THE

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[THIRD SERIES.]

ART. XXIV.—*The Wings of Pterodactyles;* (with Plate III), by Professor O. C. MARSH.

THE first Pterosaurians discovered were recognized as flying animals, but were thought to be bats. As soon as their general structure became known, they were classed with the Reptiles, although it was considered possible that their power of flight was due to feathers. Later, their bones were mistaken for those of Birds by various experienced anatomists, and others regarded them as sharing important characters with that group. Some anatomists, however, believed that the fore limbs of Pterodactyles were used for swimming, rather than for flight, and this view has found supporters within the present decade. A single fortunate discovery, made a few years since, has done much to settle the question as to the wings of Pterodactyles, as well as their mode of flight, and it is the aim of the present article to place on record some of the more important facts thus brought to light.

The specimen to be described was found in 1873, near Eichstädt, Bavaria, in the same lithographic slates that have yielded *Archæopteryx*, *Compsognathus*, and so many other Jurassic fossils known to fame. This specimen, which represents a new species of the genus *Rhamphorhynchus*, is in a remarkable state of preservation. The bones of the skeleton are nearly all in position, and those of both wings show very perfect impressions of *volant membranes* still attached to them. Moreover, the extremity of the long tail supported a separate vertical membrane, which was evidently used as a rudder in flight.

These peculiar features are well shown in Plate III, which represents the fossil in the natural size.

The discovery of this unique specimen naturally attracted much attention at the time, and many efforts were made to secure it for European museums. The writer was then at work on the toothless Pterodactyles which he had recently found in the Cretaceous of Kansas, and believing the present specimen important for his investigations, sent a message by cable to a friend in Germany, and purchased it for the museum of Yale College, where it is now deposited.

The Wing Membranes.

A careful examination of this fossil shows that the patagium of the wings was a thin smooth membrane, very similar to that of modern bats. As the wings were partially folded at the time of entombment, the volant membranes were naturally contracted into folds, and the surface was also marked by delicate striæ. At first sight, these striæ might readily be mistaken for a thin coating of hair, but on closer investigation they are seen to be minute wrinkles in the surface of the membranes, the under side of which is exposed. The wing membranes appear to have been attached in front along the entire length of the arm, and out to the end of the elongated wing finger. From this point, the outer margin curved inward and backward, to the hind foot.

The membrane evidently extended from the hind foot to near the base of the tail, but the exact outline of this portion cannot at present be determined. It was probably not far from the position assigned it in the restoration attempted in the cut given below, figure 2. The attachment of the inner margin of the membrane to the body was doubtless similar to that seen in bats and flying squirrels.

In front of the arm, there was likewise a fold of the skin extending probably from near the shoulder to the wrist, as indicated in figure 2. This fold enclosed a peculiar bone (pteroid), the nature and function of which will be discussed below in considering the osteology of this part of the skeleton.

The Caudal Membrane.

The greater portion of the tail of this specimen was free, and without volant attachments. The distal extremity, however, including the last sixteen short vertebræ, supported a vertical membrane, which is shown in Plate III, and also in the woodcut, figure 1. This peculiar caudal appendage was of somewhat greater thickness than the patagial membrane of the wings. It was rhomboid in outline, and its upper and lower

portions were slightly unequal in form and size. The upper part was kept in position by a series of spines, sent off one from near the middle of each vertebral centrum, and thus clearly representing neural spines. The lower half also was strengthened by similar spines, which descended from near the junction of the vertebræ, and hence were homologous with chevron bones. These spines were cartilaginous, and flexible, but sufficiently firm in texture to keep the membrane in an upright position.

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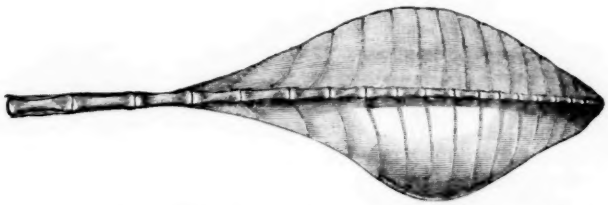

FIGURE 1.—Caudal extremity of *Rhamphorhynchus phyllurus*, Marsh; natural size. Seen from the left side.

THE SCAPULAR ARCH.

The osteology of the scapular arch and wings of Pterodactyles involves many interesting points, some of which have been discussed by anatomists from Cuvier to those of the present day, but with little agreement of opinion. The cause of this diversity of opinion is mainly due to the fact that the specimens examined have been either too small or too imperfect for accurate determination of their more obscure parts. Fortunately, the museum of Yale College has among its specimens of Cretaceous Pterodactyles (some 600 in all), quite a number with the scapular arch and wing-bones nearly perfect, and in position. These specimens were nearly all of gigantic size, having in life a spread of wings from fifteen to twenty feet. They were also destitute of teeth, and belong to the order *Pteranodontia*. Probably their great size induced special modifications of the scapular arch, which is here far more complicated than in any other members of the group.

In the Jurassic Pterodactyles, the scapula is usually bird-like in general form and proportions, the upper or distal extremity being free and compressed. This is the case in the specimen here described. The scapula and coracoid may be coössified, as in the present fossil, or remain more or less separate. No clavicles have yet been found. The sternum here shows no distinct facets for sternal ribs.

In the Cretaceous genus *Pteranodon*, and probably also in some of the other gigantic forms from deposits of this age, the

scapula and coracoid were not only solidly united, but the pectoral arch was further strengthened (1) by the ankylosis of several vertebræ, and (2) by the robust scapulæ articulating on opposite sides of the common neural spine of these vertebræ. This is virtually a repetition of the pelvic arch, on a much larger scale. The sternum also is massive, and shows well marked facets for the sternal ribs. This peculiar method of strengthening the scapular arch has not been observed in any other vertebrates.

The Wing Bones.

The three principal bones of the arm (humerus, radius, and ulna), present such similar characters in all Pterodactyles, that they need not be considered here in detail. It is important, however, to bear in mind that the ulna, although but little larger than the radius, contributes the greater share of direct support to the enormously developed wing finger, which is on the outer or ulnar side of the hand. As this position has been a question of discussion among anatomists, it may be well to state, that the writer bases his opinion upon this point on the results of an examination of the best preserved specimens in European museums, as well as nearly all known in this country. The latter specimens settle the question beyond doubt.

The views expressed by anatomists in regard to the bones of the wrist and hand of Pterodactyles are almost as various as the specimens investigated. Some of the restorations of these parts that have been published from time to time, and repeated in text books, have done much to propagate errors, and little to clear away the serious difficulties in the case. The main facts in regard to the carpus now known may be briefly stated as follows:

In all Pterodactyles, there are two principal carpal bones, placed one above the other. These sometimes show indications of being composite, but their constituent parts have not been satisfactorily determined. On the inner side of the wrist, articulating with the distal carpal, there is a smaller bone which has been called the "lateral carpal." In addition to these three bones, some American Pterodactyles have on the inner side three ossicles, which may be sesamoid bones. Two of these have been seen in a few Jurassic forms in Europe. Beside these, there is often found on the radial side of the wrist, and sometimes attached to it, a long, slender styloid bone, having a rounded articular head on its carpal extremity. This is the so-called "pteroid bone," to which allusion has already been made above. This bone and the "lateral carpal" which supports it are usually placed by anatomists on

the outer or ulnar side, but American specimens prove conclusively that they belong on the radial side.

The nature of the so-called pteroid bone has been much discussed, but without a satisfactory conclusion. After a careful study of many specimens, the writer is disposed to regard it, not as an ossified tendon, but as a part of the first digit, or thumb, which is usually considered wanting in Pterodactyles. According to this view, the "lateral carpal" would probably be the metacarpal of the same digit. In favor of this interpretation, it may be said—

(1.) That the position and structure of this appendage of the carpus correspond closely with that of the first digit in some other reptiles, for example, *Iguanodon*.

(2.) The "lateral carpal" unites both with the distal carpal and with the "pteroid" by very free, well-defined articulations.

(3.) In American specimens, the "lateral carpal" stands nearly at right angles to the wrist, and the "pteroid" is much bent near its articular end.

(4.) In no Pterodactyle known is there any remnant of a digit outside the wing finger, where the membrane might be expected to retain it.

(5.) This view would make the wing finger the fifth digit, the same to which the membrane is attached in the hind foot.

Perhaps the strongest objection against this interpretation is the number of phalanges in the respective digits of the hand. These, however, are not constant in the known Pterodactyles, and they vary much in other reptiles which have the digits highly specialized. This subject will be more fully discussed by the writer elsewhere.

According to the above interpretation, there are five digits in the hand of Pterodactyles, although not the five often given in restorations. The first digit, the elements of which have been considered, undoubtedly supported a membrane in front of the arm. The second, third, and fourth are small, and armed with claws. The large wing finger is the fifth, corresponding to the little finger of the human hand.

The metacarpal bones are much elongated in the Pterodactyles with short tails, and quite short in those, like the present specimen, that have the tail long. The metacarpal of the wing finger is always large, and robust, while those of the claw-bearing digits are usually quite slender. In *Pteranodon*, the second metacarpal is a slender thread of bone, throughout most of the length, while the third and fourth are attenuated splint bones, incomplete above.

The phalanges of the three middle digits are quite short, and the terminal ones supported sharp claws. The wing finger has four greatly elongated phalanges, the last being a styloid bone,

without a claw. This digit is well shown in the right wing represented in Plate III, and also in the restoration, given below, in figure 2.

In the restoration here attempted, the writer has endeavored to reproduce (1) the parts actually present or clearly indicated in the specimen described, and (2) those which the former seemed to require to complete the outward form in life. The

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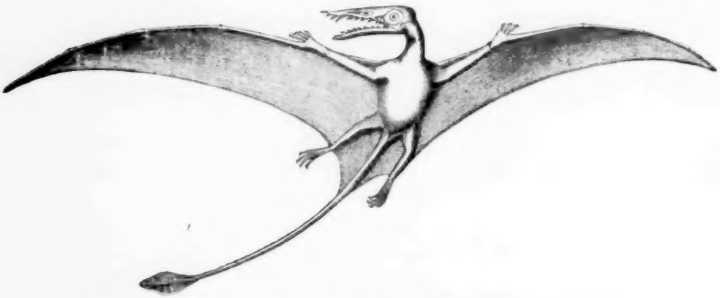

FIGURE 2.—Restoration of *Rhamphorhynchus phyllurus*, Marsh; one seventh natural size.

membrane at the base of the tail may have been somewhat less in extent, and the fold of the skin above the fore-arm either more or less developed than here represented, but the facts now known render the outlines here given more than probable. The hands are represented with the palms forward.

The present species appears to be most nearly related to *Rhamphorhynchus Gemmingi*, von Meyer, from the same geological horizon, and near the same locality. That it is quite distinct, however, is shown, aside from the difference in size, by the complete ankylosis of the scapula and coracoid, and by the fifth digit of the hind foot being well developed, and having three phalanges. In the name *Rhamphorhynchus phyllurus*, here proposed for the species, the latter designation refers to the leaf-shaped caudal appendage, which appears to be one of its most characteristic features.

For the long delay in the description of this important European specimen, the writer can only plead *l'embarras des richesses* nearer home.

Yale College, New Haven, March 14, 1882.

EXPLANATION OF PLATE III.

Rhamphorhynchus phyllurus, Marsh. Natural size.

The animal lies upon the back, and the under surfaces of the wing membranes are exposed. The caudal membrane is seen from the left side.

ART. XXV.—*On Sandstones having the grains in part Quartz Crystals;* by Rev. A. A. YOUNG, of New Lisbon, Wisconsin.

IN the Potsdam sandstone of the vicinity of New Lisbon, Wisconsin, and the St. Peters sandstone of Grant County in the same State, I have found many of the quartz grains to be true quartz crystals with polished faces. On mounting some of the grains in balsam, and then examining them with the microscope, many were found to have well-defined nuclei of varying sizes, proving that the crystalline form was produced by the finishing out of original irregular grains of the sand-bed, which are now the kernels of the crystals, thus according with observations by Mr. Sorby, as described in his Presidential address, of 1880, before the Geological Society. Some of the kernels are quite translucent, others nearly opaque, and occasionally one has a needle of rutile (?) which does not extend beyond.

ART. XXVI.—*Notes on American Earthquakes:* No. 11; by Professor C. G. ROCKWOOD, Jr., Ph.D., Princeton, N. J.

THIS article embodies such information as the author has obtained in regard to the earthquakes which occurred upon the American continent and adjacent islands during the year 1881, with notice of some earlier shocks not previously reported. Items which depend upon single sources of information have their source indicated, and if regarded as doubtful are printed in smaller type.

For assistance in collecting information the author is indebted to J. M. Batchelder, Esq., of Boston, to President J. W. Dawson, of Montreal, and to the Superintendent of the Meteorological Service at Toronto. The Monthly Weather Review of the U. S. Signal Service has also furnished valuable information.

1880.

Sept. 28.—6 P. M. Three heavy shocks on the island of Ukamok, Alaska (55° 48′ N., 155° 34′ W.); the first in north-south direction, the others west-east; 9 P. M., severe shock from W. to E. 29th, 3 A. M. and 1 P. M., heavy shocks from W. to E. "From the commencement of this phenomena, Sept. 28th, until its subsidence, Oct. 16th, there was an uninterrupted trembling motion of the earth, interspersed with heavy, subterranean, rumbling sounds. During a short trip over a portion of the island, on Sept. 29th, deep fissures, with a width of from fifteen to twenty inches were found to be very numerous." The above is reported by Mr. A. Newlander, at that time a resident of the island.—*U. S. Weath. Rev.*

Oct. 26 to 29. In regard to the earthquake shocks of these dates in Alaska (already reported, vol. xxi, p. 201), the following details of separate shocks are given by the Sitka correspondent of an Oregon paper, as are also the two succeeding items in November: Oct. 26th, 1.20 P. M., very severe shock, east–west; 2.14 P. M. and 8.46 P. M., slight shocks; 27th, 5.35 A. M., two short and sharp shocks, east–west; 9.15 P. M., sharp shock, S.W. to N.E.; 11.04 P. M. and 11.45 P. M., slight shocks, east–west. 29th, 1.05 A. M., a shock; 6.38 A. M., a sharp shock, S.E. to N.W., with rumbling noise; 11.58 P. M., three shocks, N.E. to S.W.—*U. S. Weath. Rev.*

Nov. 13.—5.28 A. M. A shock at Sitka.—*U. S. Weath. Rev.*

Nov. 14.—5.50 A. M. Two shocks at Sitka, N.E. to S.W.—*U. S. Weath. Rev.*

1881.

Jan. 1.—6.55 P. M. A shock at Red Bluff, Cal.; vibration, north–south.—*U. S. Weath. Rev.*

Jan. 5 to 7. At Bainbridge Island, W. T. (47° 42′ N., 122° 31′ W.); slight shocks at 10.56 P. M. of 5th; at 4.20 P. M. of 6th, and at 10.15 P. M. of 7th.—*U. S. Weath. Rev.*

Jan. 6.—6.25 P. M. A shock at Red Bluff, Cal.; vibrations, N.W. to S.E.—*U. S. Weath. Rev.*

Jan. 7.—6.15 A. M. Slight shock at Campo, Cal.—*U. S. Weath. Rev.*

Jan. 16.—11 P. M. Slight shock at Bainbridge Island, W. T.—*U. S. Weath. Rev.*

Jan. 20.—9.40 P. M. The shock felt in the vicinity of Bath and Brunswick, Me. (vol. xxi, p. 202), was accompanied by a rumbling noise.

Jan. 24. Three shocks felt at San Francisco and Oakland, Cal. At the former place the times were stated as 8.54 P. M., 9.15 P. M., and 11.15 P. M., and the direction N.W. to S.E. At Oakland the first and second shocks were accompanied by a low rumbling, and the direction is stated to have been S.W. to N.E.

Jan. 30.—9.45 P. M. A slight shock at Bainbridge Island, W. T.—*U. S. Weath. Rev.*

Feb. 1. At Visalia, Cal., at 4.11 P. M., three shocks lasting altogether about two seconds, and at 9.53 P. M. two more shocks. Motion, S.E. to N.W.—*U. S. Weath. Rev.*

Feb. 2.—A slight shock from north to south at Salinas City, Cal.—*U. S. Weath. Rev.*

Feb. 2.—About 4 A. M. a slight shock at Boston, Mass.—*J. M. B.*

Feb. 3.—4 A. M. A loud report and shock at Plymouth, Mass.—*J. M. B.*

Feb. 4.—Loud report and earthquake shock at Greenland and Stratham, N. H.

Feb. 12.—A shock at Portsmouth, N. H.—*J. M. B.*

Feb. 14. A slight shock at Ukiah, Cal., "about one o'clock;" direction, east–west.—*San Francisco Chronicle.*

Feb. 26.—10.55 P. M. Slight shock at Augusta, Me.—*Augusta Journal.*

March 14.—10.30 P. M. Slight shock at Bainbridge Island, W. T.; duration, thirty seconds.—*U. S. Weath. Rev.*

March 18.—9.30 P. M. Four slight shocks at Schenectady, N. Y.—*N. Y. Times.*

March 25. About 7 P. M. a slight shock at Hebron, Utah, and Pioche, Nev.

April 3.—4.52 A. M. Slight shock at Antrim, N. H.—*U. S. Weath. Rev.*

April 5.—9.30 A. M. At San Cristobal, Cuba, a slight shock in direction S.E. to N.W.

April 7. At midnight on the morning of the 7th, a shock at St. Paul's Bay, on the St. Lawrence River, Quebec.

April 10. About 2 A. M. several severe earthquake shocks were felt in Central California. The district affected was from Sacramento, on the north, to Visalia, on the south, and from the Sierra Nevada to the coast. The time is variously stated from 2 A. M. to 2.15 A. M., and the direction at most places was east-west. At Sacramento and Merced there were two, and at Watsonville four shocks.

April 20. Shocks felt at Goshen, Indiana, with heavy rumbling.

April 21.—11.30 A. M. A heavy shock, accompanied by a rumbling noise, was felt at Port Jefferson, N. Y.—*N. Y. Tribune.*

April 21. A sharp earthquake shock in the Hawaiian Islands. —*San Francisco Chronicle.*

April 23. A passenger arriving on this date at New Orleans, from Belize, British Honduras, states that a number of severe earthquake shocks had recently occurred there and along the coast of Spanish Honduras; especially one that occurred three or four weeks previous had injured houses in Belize.—*N. Y. Times.*

April 24. The New York Tribune of this date speaks of a "recent" earthquake shock in Colorado, most severe in the vicinity of the Twin Lakes. No other report of it has come to hand.

April 27.—9.10 P. M. A shock at Los Angeles, Cal.; direction, S.W. to N.E.; duration, two seconds.—*U. S. Weath. Rev.*

May 17. During the night a shock in Hayti, causing several landslips.—*Nature.*

May 18. At Contoocook, N. H., two shocks, at 12.20 A. M. and between 3 and 4 A. M.

May 19. About 9 A. M. slight shock at Lawrence, Kan.—*Lawrence Journal.*

May 27. A shock at La Salle, Ind., in the early morning, before day.

May 31.—3.20 A. M. A heavy shock at Murray Bay and vicinity, on the St. Lawrence River, sixty miles below Quebec.

June 8. "On the night of the 8th" four shocks, one quite severe, at Greytown, Nicaragua.—*J. M. B.*

June 19. In the morning a slight shock at Ottawa, Ont.

June 19.—3.35 A. M. At Newburyport, Mass., a rumbling and shaking, lasting some seconds.

June 20.—11 A. M. Two shocks, each lasting about ten seconds, at Nickerie, Dutch Guiana. They were preceded, accompanied and followed by a rumbling sound. Barometer, 30·20 inches; thermometer, 86°.—*U. S. Weath. Rev.*

June 24 and 25. Earthquake at St. Vincents, (W. I.)—*Nature.*

June 29. Earthquake in Trinidad, (W. I.)—*Nature.*

June 30.—8 A. M. A sharp shock at Campo, Cal.; direction, S.E. to N.W., preceded and accompanied by a rumbling noise.—*U. S. Weath. Rev.*

July 2.—11 P. M. A shock at San Juan, San Benito Co., Cal.—*San Francisco Chronicle.*

July 3.—2.10 A. M. A heavy earthquake at Hanford and Visalia, Cal.; vibration, west-east in both places, accompanied by subterranean noises.—*San Francisco Chronicle.*

July 5 and 7. Earthquakes in Hayti, W. I.—*Nature.*

July 31.—9.45 P. M. A slight shock at Bangor, Me., and vicinity, felt as far distant as Dover and Augusta.

Aug. 13. An earthquake at Candoba (Cordova?), Mexico.—*San Francisco Chronicle.*

Aug. 13. A shock at Contoocook, N. H., in the early morning.—*J. M. B.*

Aug. 29. A little after 11 P. M. a slight shock at Hillsboro, Ohio, and vicinity.

Aug. 30.—7 P. M. Two slight shocks at Santa Barbara, Cal.; direction, north-south.—*U. S. Weath. Rev.*

Sept. 13. Violent shocks in Maui, Hawaiian Islands.—*U. S. Weath. Rev.*

Sept. 18.—5.20 P. M. A severe shock at San Francisco, Cal.; motion, west-east; duration, five seconds; felt also slightly at Angel Island.

Sept. 25. A slight shock at Elmira, N. Y., preceding a violent storm.

Sept. 30. About 5 A. M. a severe and destructive earthquake shock occurred in the Hawaiian Islands, followed after a few minutes by a second shock somewhat less severe. The first shock was reported from Honolulu at 4.53 A. M. Light shocks were felt through that day and three moderate ones on Oct. 1st; and the lighter tremblings continued through Oct. 3d. The earthquake appears to have been most destructive in the two Konas where many buildings and cisterns were injured.

Oct 1.—1.40 A. M. A strong shock occurred at Kamouraska, Quebec, on the south side of the St. Lawrence River. It may be remarked that twice before during the year (April 7 and May 31), this same vicinity had been shaken. St. Paul's Bay and Murray Bay are nearly opposite Kamouraska.

Oct. 2.—9 A. M. A sharp shock at Campo, Cal., accompanied by a heavy rumbling sound; direction, S.E. to N.W.; duration, eight to ten seconds.—*U. S. Weath. Rev.*

Oct. 2.—1.30 P. M. A light shock at Chilcoot, Alaska.—*U. S. Weath. Rev.*

Oct. 6. A little after midnight, on the morning of this day, a slight shock was felt at Concord and Bristol, and other neighboring places in New Hampshire; direction, east-west, and accompanied by a rumbling noise.

Oct. 6.—11 A. M. A light shock at Chilcoot, Alaska.—*U. S. Weath. Rev.*

Oct. 21.—7 P. M. At Virginia City, Nev., two shocks, each lasting about two seconds; direction, S.W. to N.E. The same shock was reported from Carson City at 6.41 P. M.; direction, south–north.

Oct. 31.—1.40 A. M. A slight shock at Contoocook, Henniker, Deering and Hillsboro, N. H.

Nov. 9.—10.10 A. M. Two sharp shocks at Virginia City and Carson, Nev., from the south.

Nov. 11.—4 P. M. and Nov. 13, 11.20 P. M. Slight shocks at San Francisco, Cal.—*U. S. Weath. Rev.*

Nov. 13. An earthquake at Iquique, Chili.—*Nature.*

Nov. 15. Severe shock at noon at San José, Cal.—*N. Y. Times.*

Dec. 4.—6.30 P. M. A slight shock at Huntingdon, Quebec; direction, west–east.

Dec. 7. A heavy shock at Eureka, Nev.—*U. S. Weath. Rev.*

Dec. 16.—4 P. M. A slight shock at Dorchester, Mass.; felt only in a small district.—*J. M. B.*

Dec. 26.—11.45 P. M. A slight shock at Kingston, Jamaica.—*U. S. Weath. Rev.*

The above record includes notices of seventy-one distinct earthquakes, of which ten are in small type. They may be classified geographically as follows:

Canada,	5	West Indies,	6
New England,	14	Guiana,	1
Atlantic States,	3	Chili,	1
Mississippi Valley,	5	Hawaiian Islands,	3
Pacific Coast,	30		
Mexico and Central America,	3	Total,	71

Of the ten doubtful items, six are in New England.

In reviewing the list it will be noticed that no earthquake during the year has affected any wide extent of country, those of April 10th, in California, and Sept. 30th, in the Hawaiian Islands, being the most extensive. Also none (unless the one just mentioned) has caused any notable damage to buildings or property. It is worthy of mention also that there are only *two* items from South America (June 20, Guiana, and Nov. 13, Chili). Has the Chili-Peruvian war interfered with our usual channels of information, or caused any earthquakes there to be unreported?

Princeton, Feb. 27, 1882.

ART. XXVII.—*Notes on the Electromagnetic Theory of Light;* by J. WILLARD GIBBS, Professor of Mathematical Physics in Yale College. No. I. *On Double Refraction and the Dispersion of Colors in perfectly transparent Media.*

1. IN calculating the velocity of a system of plane waves of homogeneous light, regarded as oscillating electrical fluxes, in transparent and sensibly homogeneous bodies, whether singly or doubly refracting, we may assume that such a body is a very fine-grained structure, so that it can be divided into parts having their dimensions very small in comparison with the wave-length, each of which may be regarded as entirely similar to every other, while in the interior of each there are wide differences in electrical as in other physical properties. Hence, the average electrical displacement in such parts of the body may be expressed as a function of the time and the coördinates of position by the ordinary equations of wave-motion, while the real displacement at any point will in general differ greatly from that represented by such equations.

It is the object of this paper to investigate the velocity of light in perfectly transparent media which have not the property of circular polarization in a manner which shall take account of this difference between the real displacements and those represented by the ordinary equations of wave-motion. We shall find that this difference will account for the dispersion of colors, without affecting the validity of the laws of Huyghens and Fresnel for double refraction with respect to light of any one color.

In this investigation, it is assumed that the electrical displacements are *solenoidal*, or, in other words, that they are such as not to produce any change in electrical density. The disturbance in the medium is treated as consisting entirely of such electrical displacements and fluxes, and not complicated by any distinctively magnetic phenomena. It might therefore be more accurate to call the theory (as here developed) *electrical* rather than *electromagnetic*. The latter term is nevertheless retained in accordance with general usage, and with that of the author of the theory.

Since the velocity which we are seeking is equal to the wave-length divided by the period of oscillation, the problem reduces to finding the ratio of these quantities, and may be simplified in some respects by supposing that we have to do with a system of stationary waves. That the relation of the wave-length and the period is the same for stationary as for progressive waves is evident from the consideration that a sys-

tem of stationary waves may be formed by two systems of progressive waves having opposite directions.

2. Let x, y, z be the rectangular coördinates of any point in the medium, which with the system of waves we may regard as indefinitely extended, and let $\xi+\xi'$, $\eta+\eta'$, $\zeta+\zeta'$ be the components of electrical displacement at that point at the time t; ξ, η, ζ being the average values of the components of electrical displacement at that time in a wave-plane passing through the point. Then ξ, η, ζ, ξ', η', ζ' are perfectly defined quantities, of which ξ, η, ζ are connected with x, y, z, and t by the ordinary equations of wave-motion, while each of the quantities ξ', η', ζ' has always zero for its average value in any wave-plane. We may call ξ, η, ζ the components of the *regular* part of the displacement, and ξ', η', ζ' the components of the *irregular* part of the displacement. In like manner, the differential coefficients of these quantities with respect to the time, $\dot{\xi}$, $\dot{\eta}$, $\dot{\zeta}$, $\dot{\xi}'$ $\dot{\eta}'$, $\dot{\zeta}'$, may be called respectively, the components of the regular part of the flux, and the components of the irregular part of the flux.

Let the whole space be divided into elements of volume Dv, very small in all dimensions in comparison with a wave-length, but enclosing portions of the medium which may be treated as entirely similar to one another, and therefore not infinitely small. Thus a crystal may be divided into elementary parallelopipeds, all the vertices of which are similarly situated with respect to the internal structure of the crystal. Amorphous solids and liquids may not be capable of division into equally small portions of which physical similarity can be predicated with the same rigor. Yet we may suppose them capable of a division substantially satisfying the requirements.

From these definitions it follows that at any given instant the average value of each of the quantities ξ'. η', ζ' in an element Dv is zero. For the average value in one such element must be sensibly the same as in any other situated on the same wave-plane. If this average were not zero, the average for the wave-plane would not be zero. Moreover, at any given instant, the values of ξ, η, ζ may be regarded as constant throughout any element Dv, and as representing the average values of the components of displacement in that element. The same will be true of the quantities $\dot{\xi}'$, $\dot{\eta}'$, $\dot{\zeta}'$ and $\dot{\xi}$, $\dot{\eta}$, $\dot{\zeta}$.

3. Since we have excluded the case of media which have the property of circular polarization, we shall not impair the generality of our results if we suppose that we have to do with linearly polarized light, *i. e.*, that the regular part of the displacement is everywhere parallel to the same fixed line, all cases not already excluded being reducible to this. Then, with the origin of coördinates and the zero of time suitably chosen,

the regular part of the displacement may be represented by the equations

$$\left.\begin{aligned} \xi &= \alpha \cos 2\pi \frac{u}{l} \cos 2\pi \frac{t}{p}, \\ \eta &= \beta \cos 2\pi \frac{u}{l} \cos 2\pi \frac{t}{p}, \\ \zeta &= \gamma \cos 2\pi \frac{u}{l} \cos 2\pi \frac{t}{p}, \end{aligned}\right\} \qquad (1)$$

where l denotes the wave-length, p the period of vibration, α, β, γ the maximum amplitudes of the displacements ξ, η, ζ, and u the distance of the point considered from the wave-plane which passes through the origin. Since u is a linear function of x, y, and z, we may regard these equations as giving the values of ξ, η, ζ, for a given system of waves, in terms of x, y, z, and t.

4. The components of the irregular displacement, ξ', η', ζ', at any given point, will evidently be simple harmonic functions of the time, having the same period as the regular part of the displacement. That they will also have the same phase is not quite so evident, and would not be the case in a medium in which there were any absorption or dispersion of light. It will however appear from the following considerations that in perfectly transparent media the irregular oscillations are synchronous with the regular. For, if they are not synchronous, we may resolve the irregular oscillations into two parts, of which one shall be synchronous with the regular oscillations, and the other shall have a difference of phase of one-fourth of a complete oscillation. Now, if the medium is one in which there is no absorption or dispersion of light, we may assume that the same electrical configurations may also be passed through in the inverse order, which would be represented analytically by writing $-t$ for t in the equations which give ξ, η, ζ, ξ', η', ζ', as functions of x, y, z, and t. But this change would not affect the regular oscillations, nor the synchronous part of the irregular oscillations, which depends on the cosine of the time, while the non-synchronous part of the irregular oscillations, which depends on the sine of the time, would simply have its direction reversed. Hence, by taking first one-half the sum, and secondly one-half the difference, of the original motion and that obtained by substitution of $-t$ for t, we may separate the non-synchronous part of the irregular oscillations from the rest of the motion. Therefore, the supposed non-synchronous part of the irregular displacement, if capable of existence, is at least wholly independent of the wave-motion and need not be considered by us.

We may go farther in the determination of the quantities ξ', η', ζ'. For in view of the very fine-grained structure of the medium, it will easily appear that the manner in which the general or average flux in any element Dv (represented by ξ, η, ζ) distributes itself among the molecules and intermolecular spaces must be entirely determined by the amount and direction of that flux and its period of oscillation. Hence, and on account of the superposable character of the motions which we are considering, we may conclude that the values of ξ', η', ζ' at any given point in the medium are capable of expression as linear functions of ξ, η, ζ in a manner which shall be independent of the time and of the orientation of the wave-planes and the distance of a nodal plane from the point considered, so long as the period of oscillation remains the same. But a change in the period may presumably affect the relation between ξ', η', ζ' and ξ, η, ζ to a certain extent. And the relation between ξ', η', ζ' and ξ, η, ζ will vary rapidly as we pass from one point to another within the element Dv.

5. In the motion which we are considering there occur alternately instants of no velocity and instants of no displacement. The statical energy of the medium at an instant of no velocity must be equal to its kinetic energy at an instant of no displacement. Let us examine each of these quantities, and consider the equation which expresses their equality.

6. Since in every part of an element Dv the irregular as well as the regular part of the displacement is entirely determined (for light of a given period) by the values of ξ, η, ζ, the statical energy of the element must be a quadratic function of ξ, η, ζ,—say

$$(A\xi^2+B\eta^2+C\zeta^2+E\eta\zeta+F\zeta\xi+G\xi\eta)\,Dv,$$

where A, B, etc. depend only on the nature of the medium and the period of oscillation. At an instant of no velocity, when

$$\sin 2\pi\frac{t}{p}=0, \quad \text{and} \quad \cos^2 2\pi\frac{t}{p}=1,$$

the above expression will reduce by equations (1) to

$$(A\alpha^2+B\beta^2+C\gamma^2+E\beta\gamma+F\gamma\alpha+G\alpha\beta)\cos^2 2\pi\frac{u}{l}\,Dv.$$

Since the average value of $\cos^2 2\pi\frac{u}{l}$ in an indefinitely extended space is $\frac{1}{2}$, we have for the statical energy in a unit of volume

$$S=\tfrac{1}{2}(A\alpha^2+B\beta^2+C\gamma^2+E\beta\gamma+F\gamma\alpha+G\alpha\beta). \qquad (2)$$

7. The kinetic energy of the whole medium is represented by the double volume-integral*

$$\tfrac{1}{2}\Sigma\iint\frac{(\dot\xi+\dot\xi')_1\,(\dot\xi+\dot\xi')_2}{r}\,dv_1\,dv_2$$

where dv_1, dv_2 are two infinitesimal elements of volume, $(\dot\xi+\dot\xi')_1$, $(\dot\xi+\dot\xi')_2$ the corresponding components of flux, r the distance between the elements, and Σ denotes a summation with respect to the coördinate axes. Separating the integrations, we may write for the same quantity

$$\tfrac{1}{2}\Sigma\int(\dot\xi+\dot\xi')_1\left[\int\frac{(\dot\xi+\dot\xi')_2}{r}\,dv_2\right]dv_1.$$

It is evident that the integral within the brackets is derived from $\dot\xi+\dot\xi'$ by the same process by which the potential of any mass is derived from its density. If we use the symbol *Pot* to express this relation, we may write for the kinetic energy

$$\tfrac{1}{2}\Sigma\int(\dot\xi+\dot\xi')\ \mathrm{Pot}\ (\dot\xi+\dot\xi')\,dv.$$

The operation denoted by this symbol is evidently distributive, so that $\mathrm{Pot}\,(\dot\xi+\dot\xi')=\mathrm{Pot}\,\dot\xi+\mathrm{Pot}\,\dot\xi'$. The expression for the kinetic energy may therefore be expanded into

$$\tfrac{1}{2}\Sigma\int\dot\xi\ \mathrm{Pot}\ \dot\xi\,dv+\tfrac{1}{2}\Sigma\int\dot\xi\ \mathrm{Pot}\ \dot\xi'\,dv$$
$$+\tfrac{1}{2}\Sigma\int\dot\xi'\ \mathrm{Pot}\ \dot\xi\,dv+\tfrac{1}{2}\Sigma\int\dot\xi'\ \mathrm{Pot}\ \dot\xi'\,dv.$$

But $\dot\xi'$, and therefore Pot $\dot\xi'$, has in every wave-plane the average value zero. Also $\dot\xi$, and therefore Pot $\dot\xi$, has in every wave-plane a constant value. Therefore the second and third integrals in the above expression will vanish, leaving for the kinetic energy

$$\tfrac{1}{2}\Sigma\int\dot\xi\ \mathrm{Pot}\ \dot\xi\,dv+\tfrac{1}{2}\Sigma\int\dot\xi'\ \mathrm{Pot}\ \dot\xi'\,dv, \qquad (3)$$

which is to be calculated for a time of no displacement, when

$$\dot\xi=\pm\frac{2\pi\alpha}{p}\cos 2\pi\frac{u}{l},\quad \dot\eta=\pm\frac{2\pi\beta}{p}\cos 2\pi\frac{u}{l},\quad \dot\zeta=\pm\frac{2\pi\gamma}{p}\cos 2\pi\frac{u}{l}. \qquad (4)$$

The form of the expression (3) indicates that the kinetic energy consists of two parts, one of which is determined by the regular part of the flux, and the other by the irregular part of the flux.

* The fluxes are supposed to be measured by the electromagnetic system of units. It is to be observed that the difference of opinion which has prevailed with respect to the estimation of the energy of electrical currents does not extend to such as are *solenoidal*, which may be regarded as composed of closed circuits.

8. The value of Pot $\dot{\xi}$ may be easily found by integration, but perhaps more readily by Poisson's well-known theorem, that if q is any function of position in space (as the density of a certain mass),

$$\frac{d^2 \text{Pot } q}{dx^2}+\frac{d^2 \text{Pot } q}{dy^2}+\frac{d^2 \text{Pot } q}{dz^2}=-4\pi q, \tag{5}$$

where the direction of the coördinate axes is immaterial, provided that they are rectangular. In applying this to Pot $\dot{\xi}$, we may place two of the axes in a wave-plane. This will give

$$\frac{d^2 \text{Pot } \dot{\xi}}{du^2}=-4\pi\dot{\xi}. \tag{6}$$

In a nodal plane, Pot $\dot{\xi}=0$, since $\dot{\xi}$ has equal positive and negative values in elements of volume symmetrically distributed with respect to any point in such a plane. In a wave-crest (or plane in which $\dot{\xi}$ has a maximum value), Pot $\dot{\xi}$ will also have a maximum value, which we may call K. For intermediate points we may determine its value from the consideration that the total disturbance may be resolved into two systems of waves, one having a wave-crest, and the other a nodal plane passing through the point for which the potential is sought. The maximum amplitudes of these component systems will be to the maximum amplitude of the original system as $\cos 2\pi\frac{u}{l}$ and $\sin 2\pi\frac{u}{l}$ to unity. But the second of the component systems will contribute nothing to the value of the potential. We thus obtain

$$\text{Pot } \dot{\xi}=\text{K}\cos 2\pi\frac{u}{l},$$

$$\frac{d^2 \text{Pot } \dot{\xi}}{du^2}=-\frac{4\pi^2}{l^2}\text{K}\cos 2\pi\frac{u}{l}=-\frac{4\pi^2}{l^2}\text{Pot } \dot{\xi}.$$

Comparing this with equation (6), we have

$$-\frac{4\pi^2}{l^2}\text{Pot } \dot{\xi}=-4\pi\dot{\xi},$$

$$\text{Pot } \dot{\xi}=\frac{l^2}{\pi}\dot{\xi}. \tag{7}$$

Hence, and by equations (4),

$$\tfrac{1}{2}\Sigma\int\dot{\xi}\,\text{Pot }\dot{\xi}\,dv=\frac{l^2}{2\pi}\Sigma\int\dot{\xi}^2\,dv=\frac{2\pi l^2}{p^2}(\alpha^2+\beta^2+\gamma^2)\int\cos^2 2\pi\frac{u}{l}\,dv.$$

The kinetic energy of the regular part of the flux is therefore, for each unit of volume,

$$T=\frac{\pi l^2}{p^2}(\alpha^2+\beta^2+\gamma^2). \qquad (8)$$

9. With respect to the kinetic energy of the irregular part of the flux, it is to be observed that, since $\dot{\xi}'$, $\dot{\eta}'$, $\dot{\zeta}'$ have their average values zero in spaces which are very small in comparison with a wave-length, the integrations implied in the notations Pot $\dot{\xi}'$, Pot $\dot{\eta}'$, Pot $\dot{\zeta}'$ may be confined to a sphere of a radius which is small in comparison with a wave-length. Since within such a sphere $\dot{\xi}'$, $\dot{\eta}'$, $\dot{\zeta}'$ are sensibly determined by the values of $\dot{\xi}$, $\dot{\eta}$, $\dot{\zeta}$ at the center of the sphere, which is the point for which the value of the potentials are sought, Pot $\dot{\xi}'$, Pot $\dot{\eta}'$, Pot $\dot{\zeta}'$ must be functions—evidently linear functions—of $\dot{\xi}$, $\dot{\eta}$, $\dot{\zeta}$; and $\dot{\xi}'$ Pot $\dot{\xi}'$, $\dot{\eta}'$ Pot $\dot{\eta}'$, $\dot{\zeta}'$ Pot $\dot{\zeta}'$ must be quadratic functions of the same quantities. But these functions will vary with the position of the point considered with reference to the adjacent molecules.

Now the expression for the kinetic energy of the irregular part of the flux,

$$\tfrac{1}{2}\Sigma f\dot{\xi}'\,\mathrm{Pot}\,\dot{\xi}'\,dv,$$

indicates that we may regard the infinitesimal element dv as having the energy (due to this part of the flux)

$$\tfrac{1}{2}\Sigma\dot{\xi}'\,\mathrm{Pot}\,\dot{\xi}'\,dv.$$

Let us consider the energy due to the irregular flux which will belong to the above defined element Dv, which is not infinitely small, but which has the advantage of being one of physically similar elements which make up the whole medium. The energy of this element is found by adding the energies of all the infinitesimal elements of which it is composed. Since there are quadratic functions of the quantities $\dot{\xi}$, $\dot{\eta}$, $\dot{\zeta}$, which are sensibly constant throughout the element Dv, the sum will be quadratic function of $\dot{\xi}$, $\dot{\eta}$, $\dot{\zeta}$, say

$$(A'\dot{\xi}^2+B'\dot{\eta}^2+C'\dot{\zeta}^2+E'\dot{\eta}\dot{\zeta}+F'\dot{\zeta}\dot{\xi}+G'\dot{\xi}\dot{\eta})\,Dv,$$

which will therefore represent the energy of the element Dv due to the irregular flux. The coefficients A′, B′, etc., are determined by the nature of the medium and the period of oscillation. They will be constant throughout the medium, since one element Dv does not differ from another.

This expression reduces by equations (4) to

$$\frac{4\pi^2}{p^2}(A'\alpha^2+B'\beta^2+C'\gamma^2+E'\beta\gamma+F'\gamma\alpha+G'\alpha\beta)\cos^2 2\pi\frac{u}{l}\,Dv.$$

The kinetic energy of the irregular flux in a unit of volume is therefore

$$T'=\frac{2\pi^2}{p^2}(A'\alpha^2+B'\beta^2+C'\gamma^2+E'\beta\gamma+F'\gamma\alpha+G'\alpha\beta.) \qquad (9)$$

10. Equating the statical and kinetic energies, we have

$$\tfrac{1}{2}(A\alpha^2+B\beta^2+C\gamma^2+E\beta\gamma+F\gamma\alpha+G\alpha\beta)$$
$$=\frac{\pi l^2}{p^2}(\alpha^2+\beta^2+\gamma^2)$$
$$+\frac{2\pi^2}{p^2}(A'\alpha^2+B'\beta^2+C'\gamma^2+E'\beta\gamma+F'\gamma\alpha+G'\alpha\beta). \qquad (10)$$

The velocity (V) of the corresponding system of progressive waves is given by the equation

$$V^2=\frac{l^2}{p^2}=\frac{1}{2\pi}\frac{A\alpha^2+B\beta^2+C\gamma^2+E\beta\gamma+F\gamma\alpha+G\alpha\beta}{\alpha^2+\beta^2+\gamma^2}$$
$$-\frac{2\pi}{p^2}\frac{A'\alpha^2+B'\beta^2+C'\gamma^2+E'\beta\gamma+F'\gamma\alpha+G'\alpha\beta}{\alpha^2+\beta^2+\gamma^2}. \qquad (11)$$

If we set

$$a=\frac{1}{2\pi}A-\frac{2\pi}{p^2}A', \qquad b=\frac{1}{2\pi}B-\frac{2\pi}{p^2}B', \qquad \text{etc.}, \qquad (12)$$

and

$$\rho^2=\alpha^2+\beta^2+\gamma^2,$$

the equation reduces to

$$V^2=\frac{a\alpha^2+b\beta^2+c\gamma^2+e\beta\gamma+f\gamma\alpha+g\alpha\beta}{\rho^2}. \qquad (13)$$

For a given medium and light of a given period, the coefficients a, b, etc., are constant.

This relation between the velocity of the waves and the direction of oscillation is capable of a very simple geometrical expression. Let r be the radius vector of the ellipsoid

$$ax^2+by^2+cz^2+eyz+fzx+gxy=1. \qquad (14)$$

Then

$$\frac{1}{r^2}=\frac{ax^2+by^2+cz^2+eyz+fzx+gxy}{r^2}.$$

If this radius is drawn parallel to the electrical oscillations, we shall have

$$\frac{x}{r}=\frac{\alpha}{\rho}, \qquad \frac{y}{r}=\frac{\beta}{\rho}, \qquad \frac{z}{r}=\frac{\gamma}{\rho},$$

and

$$V^2=\frac{1}{r^2}. \qquad (15)$$

That is, the wave-velocity for any particular direction of oscillation is represented in the ellipsoid by the reciprocal of the radius vector which is parallel to that direction.

11. This relation between the wave-length, the period, and the direction of vibration, must hold true not only of such vibrations as actually occur, but also of such as we may imagine to occur under the influence of constraints determining the direction of vibration in the wave-plane. The directions of the natural or unconstrained vibrations in any wave-plane may be determined by the general mechanical principle that if the type of a natural vibration is infinitesimally altered by the application of a constraint, the value of the period will be stationary.* Hence, in a system of stationary waves, such as we have been considering, if the direction of an unconstrained vibration is infinitesimally varied in its wave-plane by a constraint, while the wave-length remains constant, the period will be stationary. Therefore, if the direction of the unconstrained vibration is infinitesimally varied by constraint, and the period remains rigorously constant, the wave-length will be stationary. Hence, if we make a central section of the above described ellipsoid parallel to any wave-plane, the directions of natural vibration for that wave-plane will be parallel to the radii vectores of stationary value in that section, viz., to the axes of the ellipse, when the section is elliptical, or to all radii, when the section is circular.

12. For light of a single period, our hypothesis has led to a perfectly definite result, our equations expressing the fundamental laws of double refraction as enunciated by Fresnel. But if we ask how the velocity of light varies with the period, that is, if we seek to derive from the same equations the laws of the dispersion of colors, we shall not be able to obtain an equally definite result, since the quantities A, B, etc., and A′, B′, etc., are unknown functions of the period. If, however, we make the assumption, which is hardly likely to be strictly accurate, but which may quite conceivably be not far removed from the truth, that the manner in which the general or average flux in any small part of the medium distributes itself among the molecules and intermolecular spaces is independent of the period, the quantities A, B, etc., and A′, B′, etc., will be constant, and we obtain a very simple relation between V and p, which appears to agree tolerably well with the results of experiment.

If we set

$$H=\frac{A\alpha^2+B\beta^2+C\gamma^2+E\beta\gamma+F\gamma\alpha+G\alpha\beta}{\rho^2}, \qquad (16)$$

* See Rayleigh's *Theory of Sound*, vol. i, p. 84.

and $$H'=\frac{A'\alpha^2+B'\beta^2+C'\gamma^2+E'\beta\gamma+F'\gamma\alpha+G'\alpha\beta}{\rho^2}, \qquad (17)$$

our general equation (11) becomes

$$V^2=\frac{H}{2\pi}-\frac{2\pi H'}{p^2}, \qquad (18)$$

where H and H′ will be constant for any given direction of oscillation, when A, B, etc., and A′, B′, etc., are constant. If we wish to introduce into the equation the absolute index of refraction (n) and the wave-length in vacuo (λ) in place of V and p, we may divide both sides of the equation by the square of the constant (k) representing the velocity of light in vacuo. Then, since

$$\frac{V}{k}=\frac{1}{n}, \quad \text{and} \quad kp=\lambda,$$

our equation reduces to

$$\frac{1}{n^2}=\frac{H}{2\pi k^2}-\frac{2\pi H'}{\lambda^2}, \qquad (19)$$

It is well known that the relation between n and λ may be tolerably well but by no means perfectly represented by an equation of this form.

13. If we now give up the presumably inaccurate supposition that A, B, etc., and A′, B′, etc., are constant, equation (19) will still subsist, but H and H′ will not be constant for a given direction of oscillation, but will be functions of p, or, what amounts to the same, of λ. Although we cannot therefore use the equation to derive *a priori* the relation between n and λ, we may use it to derive the values of H and H′ from the empirically determined relation between n and λ. To do this, we must make use again of the general principle that an infinitesimal variation in the type of a vibration, due to a constraint, will not affect the period. If we first consider a certain system of stationary waves, then a system in which the wave-length is greater by an infinitesimal dl (the direction of oscillation remaining the same,) the period will be increased by an infinitesimal dp, and the manner in which the flux distributes itself among the molecules and intermolecular spaces will presumably be infinitesimally changed. But if we suppose that in the second system of waves there is applied a constraint compelling the flux to distribute itself in the same way among the molecules and intermolecular spaces as in the first system, (so that ξ', η', ζ' shall be the same functions as before of ξ, η, ζ,—a supposition perfectly compatible with the fact that the values of ξ, η, ζ are changed,) this constraint, according to the principle cited, will not affect the period of oscilla-

tion. Our equations will apply to such a constrained type of oscillation, and A, B, etc., and A′, B′, etc., and therefore H and H′, will have the same values in the last described system of waves as in the first system, although the wave-length and the period have been varied. Therefore, in differentiating equation (18), which is essentially an equation between l and p, or its equivalent (19), we may treat H and H′ as constant. This gives

$$-\frac{2}{n^3}\frac{dn}{d\lambda}=\frac{4\pi \mathrm{H}'}{\lambda^2}.$$

We thus obtain the values of H′ and H:

$$\mathrm{H}'=-\frac{\lambda^2}{2\pi n^3}\frac{dn}{d\lambda}, \qquad \mathrm{H}=\frac{2\pi k^2}{n^2}-\frac{2\pi k^2\lambda}{n^3}\frac{dn}{d\lambda}. \tag{20}$$

By determining the values of H and H′ for different directions of oscillation, we may determine the values of A, B, etc., and A′, B′, etc.

By means of these equations, the ratios of the statical energy (S), the kinetic energy due to the regular part of the flux (T), and the kinetic energy due to the irregular part of the flux (T′), are easily obtained in a form which admits of experimental determination. Equations (8) and (9) give

$$\mathrm{T}=\frac{\pi\rho^2 l^2}{p^2}, \qquad \mathrm{T}'=\frac{2\pi^2 \mathrm{H}'\rho^2}{p^2}.$$

Therefore, by (20),

$$\frac{\mathrm{T}'}{\mathrm{T}}=\frac{2\pi \mathrm{H}'}{l^2}=\frac{2\pi \mathrm{H}' n^2}{\lambda^2}=-\frac{\lambda}{n}\frac{dn}{d\lambda}=-\frac{d\log n}{d\log\lambda}. \tag{21}$$

$$\frac{\mathrm{S}}{\mathrm{T}}=\frac{\mathrm{T}+\mathrm{T}'}{\mathrm{T}}=1+\frac{\mathrm{T}'}{\mathrm{T}}=\frac{d\log\lambda-d\log n}{d\log\lambda}=\frac{d\log l}{d\log\lambda}. \tag{22}$$

$$\frac{\mathrm{T}'}{\mathrm{S}}=-\frac{d\log n}{d\log l}. \tag{23}$$

Since S, T, and T′ are essentially positive quantities, their ratios must be positive. Equation (21) therefore requires that the index of refraction shall increase as the period or wave-length in vacuo diminishes. Experiment has shown no exceptions to this rule, except such as are manifestly attributable to the absorption of light.

14. It remains to consider the relations between the optical properties of a medium and the planes or axes of symmetry which it may possess. If we consider the statical energy per unit of volume (S) and the period as constant, we may regard equation (2) as the equation of an ellipsoid, the radii vectores of which represent in direction and magnitude the amplitudes of systems of waves having the same statical energy. In like manner, if we consider the kinetic energy of the irregular part of the flux per unit of volume (T′) and the period as constant,

we may regard equation (9) as the equation of an ellipsoid, the radii vectores of which represent in direction and magnitude the amplitudes of systems of waves having the same kinetic energy due to the irregular part of the flux. These ellipsoids, which we may distinguish as the ellipsoids (A, B, etc.) and (A′, B′, etc.), as well as the ellipsoid before described, which we may call the ellipsoid (*a*, *b*, etc.), must be independent in their form and their orientation of the directions of the axes of coördinates, being determined entirely by the nature of the medium and the period of oscillation. They must therefore possess the same kind of symmetry as the internal structure of the medium.

If the medium is symmetrical about a certain axis, each ellipsoid must have an axis parallel to that. If the medium is symmetrical with respect to a certain plane, each ellipsoid must have an axis at right angles to that plane. If the medium after a revolution of less than 180° about a certain axis is then equivalent to the medium in its first position, or symmetrical with it with respect to a plane at right angles to that axis, each ellipsoid must have an axis of revolution parallel to that axis. These relations must be the same for light of all colors, and also for all temperatures of the medium.

15. From these principles, we may infer the optical characteristics of the different crystallographic systems.

In crystals of the isometric system, as in amorphous bodies, the three ellipsoids reduce to spheres. Such media are optically isotropic, at least so far as any properties are concerned which come within the scope of this paper.

In crystals of the tetragonal or hexagonal systems, the three ellipsoids will have axes of rotation parallel to the principal crystallographic axis. Since the ellipsoid (*a*, *b*, etc.) has but one circular section, there will be but one optic axis, which will have a fixed direction.

In crystals of the orthorhombic system, the three ellipsoids will have their axes parallel to the rectangular crystallographic axes. If we take these directions for the axes of coördinates, E, F, G, E′, F′, G′, *e*, *f*, *g* will vanish and equation (13) will reduce to

$$V^2=\frac{a\alpha^2+b\beta^2+c\gamma^2}{\rho^2}.$$

If the coördinate axes are so placed that

$$a>b>c,$$

the optic axes will lie in the X-Z plane, making equal angles φ with the axis of Z, which may be determined by the equation

$$\tan^2\varphi=\frac{a-b}{b-c}=\frac{p^2(A-B)-4\pi^2(A'-B')}{p^2(B-C)-4\pi^2(B'-C')}.$$

To get a rough idea of the manner in which φ varies with the period, we may regard A, B, C, A′, B′, C′ as constant in this equation.

But since the lengths of the axes of the ellipsoid (a, b, etc.) vary with the period, it may easily happen that the order of the axes with respect to magnitude is not the same for all colors. In that case, the optic axes for certain colors will lie in one of the principal planes, and for other colors in another. For the color at which the change takes place, the two optic axes will coincide. The differential coefficient $\frac{d\varphi}{dp}$ becomes infinitely great as the optic axes approach coincidence.

In crystals of the monoclinic system, each of the three ellipsoids will have an axis perpendicular to the plane of symmetry. We may choose this direction for the axis of X. Then F, G, F′, G′, f, g, will vanish and equation (13) will reduce to

$$V^2=\frac{a\alpha^2+b\beta^2+c\gamma^2+e\beta\gamma}{\rho^2}.$$

The angle θ made by one of the axes of the ellipsoid (a, b, etc.) in the plane of symmetry with the axis of Y and measured toward the axis of Z is determined by the equation

$$\tan 2\theta=\frac{e}{c-b}=\frac{p^2\,\mathrm{E}-4\pi^2\,\mathrm{E}'}{p^2\,(\mathrm{C}-\mathrm{B})-4\pi^2\,(\mathrm{C}'-\mathrm{B}')}.$$

To get a rough idea of the *dispersion* of the axes of the ellipsoid (a, b, etc.) in the plane of symmetry, we may regard B, C, E, B′, C′, E′, as constant in this equation, and suppose the axis of Y so placed as to make E vanish.

It is evident that in this system the plane of the optic axes will be fixed, or will rotate about one of the lines which bisect the angles made by the optic axes, according as the mean axis of the ellipsoid (a, b, etc.) is perpendicular to the plane of symmetry or lies in that plane. In the first case the *dispersion* of the two optic axes will be unequal. The same crystal, however, with light of different colors, or at different temperatures, may afford an example of each case.

In crystals of the triclinic system, since the ellipsoids (A, B, etc.) and (A′, B′, etc.) are determined by considerations of a different nature, and there are no relations of symmetry to cause a coincidence in the directions of their axes, there will not in general be any such coincidence. Therefore the three axes of the ellipsoid (a, b, etc.), that is, the two lines which bisect the angles of the optic axes and their common normal, will vary in position with the color of the light.

16. It appears from this foregoing discussion that by the electromagnetic theory of light we may not only account for the dispersion of colors (including the dispersion of the lines

which bisect the angles of the optic axes in doubly refracting media), but may also obtain Fresnel's laws of double refraction for every kind of homogeneous light *without neglect of the quantities which determine the dispersion of colors.*

But a closer approximation than that of this paper will be necessary to explain the phenomena of circularly polarizing media, which depend on very minute differences of wave-velocity, represented perhaps by a few units in the sixth significant figure of the index of refraction. That the degree of approximation which will give the laws of circular and elliptic polarization will not add any terms to the equations of this paper, except such as vanish for media which do not exhibit this phenomenon, will be shown in another number of this Journal.

ART. XXVIII.—*The "Timber Line;"* by HENRY GANNETT.

IN Dr. Rothrock's valuable report on botany, recently published by the "Surveys West of the 100th Meridian," the author quotes Dr. Engelmann's statement that "there is little or no increase in altitude in the timber line toward the equator, in our western hemisphere, south of the 41st parallel of north latitude."

This statement is approximately true regarding the Rocky Mountains, owing, however, not to any general principle, but to what may be termed an accident of topography. Even here a decided rise is observable from 41° to 39° of latitude. In the Sierra Nevada, the Basin and Wahsatch Ranges, the statement does not hold good, the timber line rising rapidly as the latitude decreases. Again, on the volcanic peaks of the Mexican plateau, the timber line is higher by several thousands of feet than it is anywhere in the United States.

Barring the prohibitive circumstances of absence of soil and moisture, the height of the timber line is purely a question of temperature. The latter is a function of the latitude, the elevation, and the mass, of the country in the neighborhood. A great mass of country, if raised to a considerable height above the sea, as in the case of the great Cordilleran plateau of the West, carries up with it, to a certain extent, the isothermals. A glance at Mr. Schott's admirable isothermal charts amply illustrates this general fact. Washington, D. C., has a mean annual temperature of 55° Fah., while Denver, Col., a fraction of a degree farther north, and at an elevation of 5,300 feet, has a mean temperature, not of 37°, as the height might indicate, but of 49°.

Therefore, in considering the height of the timber line, we must regard the mountain ranges in connection with the plateaus upon which they stand, their latitudes, heights and masses, or what, in a measure, sums up these three, their temperatures, as it is by these that its height is determined.

Looking at the subject from this point of view, a fair comparison may be instituted between the timber line in different latitudes and on different ranges in the same latitude.

The actual elevation above sea level of the timber line in the Cordilleras of North America ranges from 6 or 7,000 to 12,000 feet. It is lowest in the Coast and Cascade Ranges of Washington Territory, where it is at about the former figures. Following the Cascade Range southward into Oregon, the timber line rises to a height of 7,000 to 8,000 feet. It continues to increase as we trace it southward into California, being on Shasta and the neighboring mountains 8,000 feet above the sea. On the high sierras of Eastern central California, forests grow to 10,000 or 11,000 feet, while the San Bernardino and other ranges of Southern California do not reach the upper limit of forests.

Few of the ranges of Nevada reach the timber line, which is at a height of 9,000 feet in the north up to, probably, 11,000 feet in the southern part of the State.

In Arizona, probably none of the mountains reach the timber line, except the volcanic group known as the San Francisco Mountains, and the Sierra Blanca. On these the timber line is between 11,000 and 12,000 feet.

In New Mexico, it averages about 12,000 feet above sea level. There is little variation between the northern and southern parts of the territory, as the higher annual temperature of the southern part is fully compensated for by the greater altitude of the plateau in the northern part.

In Colorado, it ranges from 12,000 feet in the southern part to 11,000 in the north. It is highest in the great mass of the San Juan Mountains and in the Sangre de Cristo range, and lowest in the northern portions of the Park and Front Ranges.

In Southern Wyoming, in the Park range, which is the only one in this portion of the territory which rises above the limit of timber, this limit is at about 11,000 feet. In the Wind River and Téton Ranges, in the northwestern part of the territory, it is at an elevation of 10,000 to 11,000 feet.

In Montana and Idaho, the limit of timber is, in general, from 9,000 to 10,000 feet, being highest in the south, and lowest near the northern boundary.

In the Uinta and Wahsatch Ranges of Utah, it is about 11,000 feet, rising somewhat above this figure in the southern part of the latter range.

Thus it is seen that in the same latitude, there is a very marked difference in the height of the timber line. The less the elevation of the surrounding country, other things being equal, the lower is the limit of timber.

This suggests a farther point. The upper limit of timber must have approximately the same mean annual temperature everywhere. Of course it will differ to a slight extent in different localities, owing to difference of exposure to wind and sun, but these are mere local circumstances, not affecting the general principle. The determination of this temperature accurately is, without direct observation, of course, impossible. I have, however, computed it approximately from such data as are available, and have found tolerably close accordance among the results.

The mean annual temperature decreases about 1° Fah. for each 300 feet of abrupt ascent. In the case of Pike's Peak and Colorado Springs, where the difference of elevation is more than 8,000 feet, the change is 1° for each 295 feet. In the case of Mt. Washington and Shelburne, New Hampshire, it is 325 feet for each degree. The former case is the most favorable in every respect, and as most of our results are drawn from the western region, I have adopted, as a round number, 300 feet.

Now, if the average mean annual temperature all around the base of a mountain were known, it would be a very simple matter to determine, with some accuracy, the temperature at timber line, knowing its height and the mean height of its base. The nearest approach which can be made to this, is to assume that the station or stations at or near the base, represent the average climate, a supposition which, in many cases, is by no means correct. Using, however, in the manner indicated, such data as are at hand, I have obtained the following results:

Mountains, etc.	Height of timber line, feet.	BASE STATION.			Temperature at timber line.
		Name.	Height in feet.	Mean an. tem.	
Cunningham Pass, Colo.,	11,500	Fort Garland,	7,945	43°	31°
Mt. Lincoln, Colo.,	12,051	Fairplay,	9,965	38°	31°
Mt. Silverheels, Colo.,	11,549	"	9,965	38°	33°
Mt. Guyot, Colo.,	11,811	"	9,965	38°	32°
Mt. Powell, Colo.,	11,600	White River Agency,	6,491	45°	28°
Pike's Peak, Colo.,	11,720	Colorado Springs,	6,032	48°	29°
Gray's Peak, Colo.,	11,100	Denver,	5,244	48°	29°
Wahsatch Mts., Utah,	10,000	Salt Lake City,	4,350	52°	33°
Mt. Washington, N. H.,	4,150	Shelburne, N. H.,	700	42°	30°
Mt. Marcy, N. Y.,	4,851	Somerville, N. Y.,	412	45°	30°
" "	4,851	Plattsburgh, N. Y.,	180	44°	29°
Mt. Blackmore, Mont.,	9,550	Fort Ellis, Mont.,	4,935	44°	29°
Mt. Bridger, Mont.,	9,002	" "	4,935	44°	31°
Mt. Delano, Mont.,	8,784	" "	4,935	44°	31°

The mean of these results is 30°·4, and this is probably very near the true mean annual temperature of the timber line. The better the conditions of the determination, the nearer are the results to this mean. Mts. Blackmore and Bridger are very good cases, being on the border of the Gallatin Valley, in which Fort Ellis is situated, and but very few miles distant from the latter. Mts. Lincoln and Silverheels are also admirably situated with respect to Fairplay, but the annual temperature of the latter station is not well determined. Pike's Peak and Colorado Springs make an excellent pair of stations, being but ten miles apart, and the annual temperature at the latter place being well determined by the observations of the Signal Bureau. On the other hand, Mt. Powell and the White River Agency are widely separated by many miles of high plateaus, which may materially change the conditions of the temperature about the mountain.

Should this result, when tested by a wider range of observations, hold good, it will afford a very valuable and easily obtainable isothermal, and also enable one to estimate the height of the timber line from thermometric stations at the bases of mountain ranges.

ART. XXIX.—*Simple Method for Calibrating Thermometers;* by SILAS W. HOLMAN.

THE calibration of a thermometer by most of the methods in ordinary use is a tedious and somewhat difficult operation, and hence often neglected even in important work. For the purpose of supplying a method simple both in observation and computation, and at the same time accurate, the following process is described, which, although involving little that is novel, has not to my knowledge been used before.

First, however, it is necessary to recall to the attention of observers the fact that, without calibration correction, the readings of a thermometer having a scale of equal linear parts cannot be relied upon within one or more divisions of this scale; and that thermometer makers, knowing this, almost universally space the graduation upon the tube to correspond more or less closely with the shape of the bore, as determined by previous calibration, or by comparison with a standard (!) instrument. This practice is much more general than is ordinarily supposed, and has an important bearing upon the accuracy of the work done with such instruments. For the scale thus made is merely approximate, the dividing engine or other tool being usually changed only at such intervals as to make

the average error less than some specified amount. An inspection of these conditions will show that the calibration of such a tube and scale can be only approximate except with corrections for the inequalities of the spacing, involving an amount of labor disproportionate to the result attained. The best makers, such as Fastré, Baudin and others, have produced satisfactory thermometers graduated to equal volumes; but even these are not as reliable as instruments of less cost with a scale of equal linear parts, say of millimeters, supplemented by a calibration by the observer. The best form of tube for almost all work is one backed with white enamel, with an inverted pear-shaped bulb at the upper end of the capillary (a very important feature), and with a scale of equal arbitrary linear parts (0·7 mm. to 1 mm. is a suitable length for estimation of tenths) or of approximate degrees, for convenience, etched or engraved upon it.

Without reviewing here the methods proposed by various writers, it may be said that it has been the general plan to select beforehand upon the scale two points between which to make the calibration, this space being the "calibration unit," the errors of these points being, of course, zero. This plan has led to unnecessary complexity. Such an assumption is no more requisite in calibration after a scale has been put upon the tube, than in calibrating by the dividing-engine or micrometer before making the scale. It is obvious that the selection of these points is wholly arbitrary, and, if used at all, one or both of them may, if desirable, be chosen after the observations with the calibrating thread have been made. The choice should be made with the view to facilitating the work. Hence the use of the observed freezing and boiling points, upon which some methods are based, is most undesirable.

In the method which will now be given, either one or both of these points may be left to be selected, according to the combined conditions of length of thread employed, shape of the tube, and numerical convenience, after the observations with the thread have been made.

Let it be desired to find the calibration corrections for a given tube. Determinations which will give the errors of every 3 cm. of length will ordinarily be sufficient, but this must depend on the result sought. Separate a thread of mercury of about that length. The actual length of the thread within two or three millimeters is of no consequence whatever, and hence a suitable thread can be obtained in a very short time.

Set the thread with its lower end at or near the beginning of the graduation: call the reading * of the lower end of the

* Tenths of a division are supposed to be read by estimation.

thread l_1, and that of the upper end u_1. Move the thread less than 1 mm. and read again, finding thus l_2 and u_2. Move the thread about 1 cm. and read l_3 and u_3. Move the thread less than 1 mm. and read l_4 and u_4. So continue throughout the whole length of graduation, increasing the number of settings or repeating the whole series in reverse order and several times if the highest attainable precision is desired. This alternation between 1 mm. and 1 cm. in setting tends towards the better elimination of errors in estimation. It is not, however, essential, nor even always as well as an equal number of distributed readings. This must depend upon the skill of the observer. Avoid, as far as convenient, taking readings with an end of the thread apparently just at the line of the scale, as the width of the line, even in the best scales, is a source of considerable error.* If any point, e. g., the zero point of the graduation, has for any reason been selected as the first of which the error should be assumed zero, the settings may to advantage, though not necessarily, be made to extend each way from this.

Then u_1-l_1, u_2-l_2, etc., will give a series of lengths of the calibrating thread in all parts of the tube. Before reuniting this thread to the rest of the mercury, plot points with abscissas l_1, l_2, etc., and ordinates u_1-l_1, u_2-l_2, etc., the corresponding lengths of thread, and draw a smooth curve through the points thus obtained. This line will give a general idea of the form of the capillary bore; and, should any parts of it show considerable irregularities, the corresponding portions of the tube should at once be re-explored with the thread.

If not already done, the point A, upon the scale, to be used as the starting or reference point of the computation, should now be selected. In general the extreme ends of the tube are to be avoided, as more likely to have been rendered irregular or rapidly tapering in the process of making or joining on the bulbs. If the zero of the numbering is placed one or two centimeters from the bottom of the tube it forms a desirable starting point.

Find upon the curve the ordinate u' corresponding to the abscissa A; then with abscissa $A+u'$ find the corresponding ordinate u''; with abscissa $A+u'+u''$, find the corresponding ordinate u''', continuing to the upper limit of the graduation. If A is at a sufficient distance from the lower end of the graduation, find a similar series below the point A. These points, A, $A+u'$, $A+u'+u''$, etc., upon the graduation are separated by equal volumes of the capillary. Select any one of these as the second point of which the error is to be arbitrarily

* Some of the advantages of Neumann's method are offset by this error.

assumed as zero, and call this B. Then $A+u'+u''+ \ldots + u^{nth} = B$. There are thus n spaces of equal volume between A and B, and these correspond each to $\frac{1}{n}$th of the interval B–A. Hence the true reading (which, however, it is not necessary to compute numerically) at the point—

A	is	A
$A+u'$	"	$A+\frac{1}{n}$ (B–A)
$A+u'+u''$	"	$A+\frac{2}{n}$ (B–A)
. . . .		. . . .
B	"	B

And the error obtained by subtracting the true readings, as given in the right-hand column, from the corresponding actual readings, given in the left-hand column, at

A	is	0
$A+u'$	"	$A+u' \{A+\frac{1}{n}$ (B–A)$\} = u'-\frac{1}{n}$ (B–A)
$A+u'+u''$	"	$u'u''-+\frac{2}{n}$ (B–A)
. . . .		. . . .
B	"	0

In selecting B it might have been assumed equal to $A+u'$, thus making $n=1$. This would somewhat simplify the calculation, and would be of equal accuracy, but is objectionable from the fact that, in general, this volume would differ considerably from the average volume obtained when n has a greater value (always an integer), and the resulting series of errors would assume larger numerical values.

The errors or corrections are, for purposes of interpolation, most conveniently represented graphically by a smooth curve through points with abscissas proportional to the direct readings,

$$A,\ A+u',\ A+u'+u'', \text{ etc.},$$

and ordinates to the corresponding corrections.

Should it be necessary to increase the accuracy by a second calibration with a thread of different length, it is only necessary to take one of approximately an integral part of (B–A), and when the final curve of error is drawn, make the error at B equal to zero, distributing the difference at that point proportionally to the scale readings, among the errors at the in-

termediate points: in other words, to shift the axis of the second curve of error so that it shall make the error at B zero.

This method requires for each calibration the use of but a single thread. The computation is simple, and involves a minimum of approximation. Errors of observation are largely eliminated by the number of settings made in all parts of the tube, and by the inspection of the curve of lengths, both of which operations tend in an unusual degree to detect mistakes or any minor irregularities of the capillary. It avoids the common requirements of setting the thread exactly at certain definite points in the tube, or any approximate correction for slight errors in such setting,—two sources of considerable error and inconvenience, especially when the thread must be set near or under a line of the graduation. And, lastly, the total time of calibration for a result of given accuracy is reduced to one-half or one-third of that required by Neumann's method, the quickest and most satisfactory with which I am acquainted except that given by Pickering. The latter, described with some slight inaccuracies, at the reference noted below, is a neat application of the graphical method; and the curve of lengths of thread adopted in the method which I have described is identical with the corresponding one given by Professor Pickering, while the whole process is fully one-third shorter and somewhat more accurate. From a series of calibrations executed upon the same thermometer (one with a millimeter scale, by Baudin, of Paris), using a variety of methods, I have obtained slightly more concordant results with the proposed method than with Neumann's or Pickering's, all those possessing, however, nearly the same degree of precision, and decidedly better results with these than with any of the other existing simple methods.

Considerable aid in eliminating errors of parallax in such work is sometimes found by looking down upon the horizontal thermometer through a vertical tube having a small hole at each end. One of the cheap French microscopes with its lenses removed, and inverted in its stand, answers this purpose well.

With such a device two calibrations of the above described thermometer with threads of 3 cm. and 5 cm. respectively, each with only one series of observations, and requiring not more than one hour and a half each for completion, gave results whose average difference from each other at nine points was 0·04 mm., and the arithmetical sum of the extreme differences was 0·12 mm., a result of sufficient accuracy for any class of work of which such an instrument is capable.

For brief descriptions of methods of separating threads of mercury for calibration, reference may be made to the paper

by Russell, and the text-book by Pickering, noted below. These processes are in general use, and are safe and convenient.

Mass. Institute of Technology, Boston, Feb. 1, 1882.

References upon Calibration of Closed Thermometer Tubes.

BESSEL—Pogg. Ann., vi, 287 (1826).
RUDBERG—Pogg. Ann., ix, 353, 566.
" " xxxvii, 376 (1836).
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THIESEN—[Neumann's Meth.], Carl's Rep., xv, 285 (1879).
RUSSELL—[" " Transl. from Thiesen.] Amer. Jour. Sci., xxi, 373 (1881).
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ART. XXX.—*Physics of the Earth's Crust;* by the Rev. OSMOND FISHER, M.A., F.G.S.*

MR. OSMOND FISHER has long been known to geologists as a writer upon the higher and more difficult problems connected with the evolution of the earth's physical features. His qualifications for this kind of discussion are rare to an extreme degree, for he possesses extended knowledge of geological science considered as a category of observed facts, and unites to it both wisdom and knowledge in physical science and great skill in mathematical analysis. Such men are indeed rare, and the need for them is very urgent. For upwards of ten years papers by him have appeared in the Transactions of the Cambridge Philosophical Society and in the Journal of the Geological Society, the more important of which deal with the mechanical problems arising in inquiries as to the origin of the earth's features. These papers are almost unique, for they are attempts to apply quantitative as well as qualitative tests to the various theories which have been advanced to account for such features. The present work takes those papers, originally disjointed, and with the addition of much new matter, shapes them into a well proportioned whole, with a strong bond of connective logic running through it.

* Physics of the Earth's Crust. By the Rev. OSMOND FISHER, M.A., F.G.S. pp. 299, 8vo. London: Macmillan & Co. 1881.

The first chapter is upon Underground Temperature, and recites those observed facts which lead to the universally received opinion that the earth's interior is hot. These can be stated very briefly. They are, 1st, the observed increase of temperature as we penetrate the strata, and 2d, volcanic phenomena. These considerations are so familiar that no extended discussion is given to them. The main work of the chapter is devoted to an examination of the case presented by the artesian boring at Sperenberg in Prussia, which was believed by Prof. Mohr to lead to the inference that at the depth of only a mile the temperature ceased to augment with increasing depth. Mr. Fisher shows that these observations, though apparently anomalous at first sight, are probably not so in reality. He holds it to be a just inference that everywhere throughout the earth's external shell the temperature increases at a nearly uniform rate for each locality for 25 or 30 miles, below which horizon the increment becomes notably less rapid, and that below 160 miles at most there is no noteworthy increase. This result flows from the application of Fourier's theorem of the conduction of heat and from the amplification of that theorem by Sir William Thomson.

The second chapter has reference to the physical condition of the earth's interior. Here the conclusions are necessarily very limited. As regards the distribution of density, it is satisfactorily established that the mean density being about 5·5 and the external density about 2·65, the density of the interior approaches that of the metals, iron, silver, etc., and probably increases toward the center, but the law of increase is wholly unknown. For the purposes of subsequent discussion, it is much more essential to frame some reasonable provisional hypothesis regarding the solidity or fluidity of the earth. The physicists claim that the phenomena of tides demand extreme rigidity in the earth mass, while geological considerations demand a notable degree of interior plasticity if not liquidity. As a compromise, Mr. Fisher asks whether the demands of both parties may not be met by postulating a large solid nucleus and solid crust with an intervening layer of plastic magma. Such a construction could be accounted for (plausibly at least) by regarding the solidity of the nucleus as due to pressure and the solidity of the crust as due to cooling; while the intervening shell is nearly as hot as the nucleus and maintains the liquid condition because it is subject to a lighter pressure.

The third chapter is a quantitative investigation of the densities and pressures existing at various depths which follow from assuming 1st, the law adopted by Sartorius von Waltershausen, and 2d, the law of Laplace.

In the fourth chapter he proceeds to some of the geological

aspects of the subject. The outward appearance of the crust of the earth indicates that it has been subjected to great violence. The distortions of the crust and also the special phenomena of volcanism, demand a source of energy for their explanation. Within the earth we know of heat and gravitation as possible sources of such energy. Have these really been the agents, and if so, in what specific manner have they acted? The compression to which strata seem to have been subjected, is very generally explained by the hypothesis that the interior of the globe has contracted through secular cooling, while the crust collapsing upon the shrinking nucleus becomes wrinkled and distorted. Mr. Fisher proceeds to compute the intensity of the compressive force which would be generated in a tangent to the crust upon this supposition, and finds it to be about 830,000 tons upon the square foot. So far as intensity is concerned, there can be no doubt of the sufficiency of this pressure.

Having shown the sufficiency of this factor he then proceeds to inquire (Chapter V) whether the work has really been accomplished in this way. He begins by seeking for some measure of the inequalities of the surface; taking first the greater inequalities; the oceanic basins and continents. If the existing inequalities of the surface were leveled down and spread out they would, he estimates, form a layer over the whole earth from 9,500 to 13,000 feet thick—an under- rather than an over-estimate.

In the next chapter he proceeds to inquire how great are the inequalities which might have been produced by secular cooling. Using Thomson's adaptation of Fourier's law of cooling, he computes that even upon the most extravagant assumption the inequalities would not amount to more than a layer 900 feet thick, and upon a moderate assumption 200 feet would be nearer the mark.

So great is this discrepancy that the contractional hypothesis becomes wholly incompatible with the theory of a solid globe, and we seem compelled to conclude either (1) that the inequalities are not due to compression, or (2) some other cause than the contraction of a solid globe through mere cooling must be sought. In this chapter also, Mr. Fisher brings out some considerations which are of crushing and destructive weight against the contractional hypothesis as ordinarily conceived. Yet he regards the facts of nature to be substantial proof of the reality of great lateral compression, and he requires "a liquid or at least plastic substratum for the crust to rest on which will allow it to undergo a certain amount of lateral shift." If this be granted many things will become explicable. From a liquid or plastic substratum it would follow that the position of

rest in the crust would be approximately the position of hydrostatic equilibrium. Mr. Fisher assumes this plastic substratum henceforward through the remainder of his work.

The subsequent chapters are devoted chiefly to a consideration of the consequences which would flow from the following postulates, both of which Mr. Fisher regards as being forced upon our conviction by the nature of the facts to be explained.

1st. That lateral compression has acted upon a grand scale in developing the earth's physical features; 2d, that the earth has an inflexible crust resting upon a liquid or plastic substratum. It would be impossible in so brief a notice to do justice to the many ingenious, suggestive and valuable ideas he throws out in this discussion and only the most striking ones can be alluded to.

Mr. Fisher inclines to the following explanation of the origin of mountains. He recognizes difficulties in it but it avoids more difficulties than any other he can think of. Granting a plastic substratum of somewhat greater density than the crust, it is possible that the compression of the strata (from whatsoever cause arising) may be localized in a narrow belt or zone. At this disturbed tract, as he calls it, where the yielding takes place, the crust becomes thickened greatly. But its position of rest must observe the law of hydrostatic equilibrium or simple flotation. If by means of compression the amount of *lighter* crust-matter is increased in any locality it displaces *denser* matter in the plastic substratum. The height to which the surface of the disturbed tract will rise above the mean level will be proportional to the difference of densities of crust and substratum respectively and also to the amount of local thickening of the crust by the compression. The ratio of the density of the crust to the density of substratum he takes at about 0·905, which is not far from that of ice to water. Hence when a disturbed tract of the crust is thickened by compression much the greater part of the thickening consists in additions to the under surface of the crust and only a small part in additions to the upper surface. The bulge downward is some nine times greater than the bulge upward. Mr. Fisher's great difficulty here is in finding an adequate source of so great an amount of compression. He examines the possibilities of contraction by secular cooling and of contraction by the extravasation of volcanic effluvia and rejects both as being totally inadequate. As a last resort he suggests with much diffidence the following as a possible source, viz: the opening of fissures in the under portion of the crust, which being filled with elastic vapors are at once subjected to great horizontal compression. Although this suggestion appears at first untenable it would be simple justice to Mr. Fisher to read his exposition of it before passing

judgment upon it. The discussion cannot be briefly summarized.

In his view, volcanic energy is the motive power in producing compression; thus reversing the hypothesis of Mallett who derived volcanic energy from compression. Mr. Fisher also derives from his construction a mechanism for volcanic action, but since it is impossible to abstract his view and do it justice, the reader must be referred to the work itself.

If we may venture to sum up in a very few sentences the general tenor of this book we should say that its earlier and middle chapters show that interior contraction cannot be the source or origin of the earth's physical features. Nevertheless Mr. Fisher feels that the facts presented by the disturbed tracts of the earth's crust compel us to assume that they have been subject to great compression, and we have no alternative but to accept this as a fact plain, simple and taught clearly by Nature herself. Failing to find beneath the crust any source of compression agreeing in amount and kind with that demanded by the facts he seeks one within the crust itself. The opening of fissures in the under surface of the crust which are filled at once by elastic vapors would, he thinks, supply the requisite machinery not only of compression but also of volcanic action.

In this work Mr. Fisher has rendered extremely valuable service to the science of physical geology, and chiefly to that branch which deals with its largest and noblest problems. My own thoughts have for some years run so much in the same paths, that it would be most pleasant to speak at considerable length upon many points he has discussed, but I can advert to only a few of them. First and foremost he has rendered most effectual service in utterly destroying the hypothesis, which attributes the deformations of the strata and earth's crust, to interior contraction by secular cooling. No person, it seems to me, can sufficiently master the cardinal points of his analysis, without being convinced that this hypothesis is nothing but a delusion and a snare, and that the quicker it is thrown aside and abandoned, the better it will be for geological science.

But while he has overthrown the hypothesis of contraction by secular cooling, there are possibly a few geologists, who may feel constrained to take the ground so ably maintained by Professor LeConte, that if there has been no appreciable contraction by secular cooling, there has been some other cause of interior contraction; for contraction is, they think, absolutely necessary to explain the physical features of the earth. Mr. Fisher has touched—all too briefly it seems to me—upon an argument quite as fatal to this modified form of the theory, as the one he has so fully elaborated, and indeed, of a more direct and comprehensive character. For

the features to be explained are not such as would have been produced by contraction. The strains set up in the crust by a shrinking nucleus would be such, that for any given amount of corrugation with the axes in one direction, there must be an equal amount with the axes at right angles to that direction. The localization of mountain chains and plications in long narrow belts, with the axes of the folds all approximately parallel, with no corresponding plications at right angles to them, is an impossible result of a collapsing spherical shell. It certainly seems as if those who advocate contraction had inferred that the tangential strains set up by such a cause, would act only in two opposite directions; whereas, since they must be uniformly distributed over the entire spherical surface, they must act in every direction within a tangent plane at any point.

Mr. Fisher's postulate of a solid crust, resting upon a plastic substratum, is one which seems indispensable to any rational theory of terrestrial physics. It will hardly be questioned by any geologist. Indeed, is not the proof of it abundant and complete? Surely no one can question the fact that the vast bodies of strata deposited in all areas of maximum sedimentation have sunk bodily as rapidly as they accumulated. The Paleozoic strata of Western Europe and Eastern America, the Carboniferous and Mesozoic system of the west, were accumulated in comparatively shallow waters with the surface of deposition almost constantly near sea-level. But if they progressively sank in this way they must have displaced yielding matter beneath. How could it have been otherwise? In the face of a conclusion sustained by evidence so irrefragable, it is certainly to be hoped that no geologist will have his faith at all shaken by any purely theoretical conclusions which may have been reached by physicists in their discussions of the effects of tidal strains upon the earth. Mr. Fisher, however, proposes a very fair compromise to the physicists. He might be understood as saying to them, "give me a rigid crust resting upon a plastic substratum, and you may do what you like with the remainder." If this concession does not meet the requirements of the physicist so much the worse for the tidal argument. In truth the position of the geologist here is incomparably the stronger of the two. The plasticity of at least a thin shell next below the solid external rocks has a validity of the highest order. Reasoning or induction scarcely enter into it—it is substantially an observed fact. But the tidal argument is open to the charge of a well-known form of logical fallacy which used to be described as "proving too much." It asserts that the earth must be inconceivably rigid, otherwise we should have no tides. But we have the tides and nobody seriously believes in the transcendental rigidity.

Perhaps the best feature of Mr. Fisher's book is the skillful use he makes of a direct consequence or corollary of a plastic substratum. The elevations and depressions of the different portions of the earth's surface, he argues, are, on the theory of a plastic substratum, determined by hydrostatic laws alone. Rigidity can have but slight influence upon them. For rigidity is a quantity which relatively decreases as the magnitudes of the masses involved increase. In the continental and oceanic areas, in great plateaus and mountain systems, rigidity is a vanishing quantity, and even in individual ridges of grand proportions it probably has no great value, as compared with the forces which produce elevations and depressions. The profiles of the earth therefore become, in their broader features, simply those which are due to flotation. The higher portions float higher because they are less dense; the depressed portions sink deeper because they are more dense. The manner in which Mr. Fisher treats this very obvious consequence of a plastic substratum is most admirable. It is one of the fundamental doctrines of his book.*

Having proceeded thus far it is somewhat surprising that Mr. Fisher did not advance one step farther. Elevations and depressions (considered as actual movements) mean one of two things. (1.) Either the quantity of matter underlying the vertically moving surface, has been increased or diminished, or else (2.) the volume of underlying matter has increased or diminished. The change is either a local change of mass or a change of density. The contractional hypothesis is an attempt to obtain an increase of mass in elevated regions, and a constant mass in the depressed regions. It has signally failed and so must any theory of this purport fail; for later investigations are more and more firmly establishing the fact, that elevated regions are not regions of greater mass nor are depressed regions, regions of less mass; but the contrary. And even if the conclusion sought were for a moment supposed to be true, the continents and great plateaus, as Mr. George Darwin has recently shown, could not be sustained without a transcendentally rigid globe; much less could they with a very rigid globe ever have been pushed up. It only remains to seek the required solution in causes which will produce *local* changes of density. This dilemma is by no means sought. We are apparently driven to it by the most inexorable of logical necessities. Here the question subdivides. Shall we assume that these

* I have long been convinced that this doctrine must form an important part of any true theory of the earth's evolution. In an unpublished paper I have used the terms isostatic and isostacy to express that condition of the terrestrial surface which would follow from the flotation of the crust upon a liquid or highly plastic substratum;—different portions of the crust being of unequal density. Isobaric would have been a preferable term, but it is preoccupied in hypsometry.

changes of density are all so many varying degrees of increased density, or shall we boldly assume that local expansion is a cause of upheaval and the reverse a cause of depression? Mr. Fisher has shown the difficulty which attends the former view. There is no difficulty in supposing that the crust and subcrust to a depth of 100 to 150 miles (not the true interior be it observed) may have contracted its volume. It may have done so unequally in different portions. But the differences of elevation in various parts of the earth are too great to admit of reasonable explanation in this way. Still more serious and even fatal is the fact that some regions have alternately risen, sunk and risen again. Calling this "columnar" contraction to distinguish it from nuclear contraction, it must be said that columnar contraction alone cannot explain the facts. I see no resource but to call to our aid columnar expansion. It will at once be objected that physical science furnishes us no warrant in the known processes of nature for such an assumption. Very true. Let us all go to work therefore, and try to find a warrant for it.

C. E. DUTTON.

ART. XXXI.—*Notes on Physiological Optics*, No. III; by W. LECONTE STEVENS.

1. THEORY OF ASSOCIATED MUSCULAR ACTION.

IN previous articles[1] it has been shown that the current theory of binocular perspective applied to the stereoscope is not only incapable of accounting for many observed facts but unsatisfactory even when the visual lines are convergent; that the apparent position of points in the stereoscopic field of view cannot be determined by any mathematical formula or accurately represented by diagram; and that this impossibility is due to physiological conditions attendant upon the abnormal use of the eyes.

That convergence of visual lines should be deemed a necessary condition in binocular vision, or, if not necessary, at least invariably present, and that it should be assumed, either expressly or implicitly, in most, if not all, of our text-books, is doubtless due to the fact that theoretically the stereoscope is intended to reproduce as nearly as possible the conditions attendant upon normal binocular vision of the objects pictured. Sir David Brewster gave especial emphasis to the theory that every point in the field of view is determined by triangulation with visual lines, elaborating this idea first in an article,[2] published in 1844, "On the Knowledge of Distance given by

Binocular Vision," and subsequently in his book on the Stereoscope,[3] published in 1856. Professor W. B. Rogers contributed to this Journal in 1855 and 1856 a series of most interesting articles on Binocular Vision,[4] in which he determined mathematically what should be the form of the resultant curve when images of dissimilar lines are binocularly combined, each point of each curve being determined by intersection of visual lines. I have performed most of Professor Rogers' experiments successfully with optic divergence. The same remark applies to those of Professor LeConte, wherever the combination is effected by diminution of the convergence that would be natural in ordinary vision.

In 1862, Professor C. F. Himes, at that time of Troy University, published an article[5] in which he criticised Brewster's theory and called attention to the modification necessitated by the possibility of stereoscopic vision with divergence of visual lines. This article, which was sent me by the author immediately after the publication of my paper in November last in this Journal, establishes his priority in this country in discovering the possibility of stereoscopic vision by this method. In 1861 two Germans, Rollet and Becker, published a method of combining similar images by optic divergence, the possibility of doing this having been already mentioned by Burckhardt.[6] Professor Himes' observations were made without any knowledge of what had just been accomplished in Germany. My own discovery was likewise independent; finding however that others had preceded me, and that prisms had been often employed to test the external rectus muscles. I claimed nothing on this ground, but devoted my attention to the analysis of vision by optic divergence; for on this little had been written. My rejection of Brewster's theory and the expression of my conviction that in associated muscular action is to be found the explanation of what has generally been referred to intersection of visual lines, was publicly made in my papers, read June 6th and Aug. 19th, 1881, the latter having since been published in this Journal. It is a source of satisfaction now to find, in the London Lancet, of Oct. 22d, and Dec. 31st, 1881, two able papers written by Brigade Surgeon Tyler Oughton, of the English army, who with no knowledge of what had been expressed by me, reaches conclusions closely akin to my own, substituting for the current theory that of "muscular consent," and rejecting the theory of corresponding retinal points.

It is but right to add that this theory was virtually stated by Professor Huxley[12] in 1868, and in such a way as quite plainly to indicate its applicability to the phenomena of optic divergence. That he should have been satisfied with a brief statement that has passed almost unnoticed, instead of elabo-

rating it in refutation of Brewster's theory, was probably due to the fact that the fallacy and popularity of the latter, in its application to the stereoscope, had not been brought especially to his attention.

2. Relation between Different Elements of Binocular Perspective.

The fact that in all stereoscopic vision there is necessarily an interruption of the usual relation between the axial and focal adjustments of the eyes was first noticed by Professor W. B. Rogers, who makes however no reference to the production of any disturbance of perspective, as noticed by myself. In the articles already published it has been shown that even when the stereograph is so constructed as to exclude to the utmost the ordinary elements of perspective, there are left still three to consider. These are—

I. The optic angle, positive or negative, enclosed by the visual lines and interpreted through the sensation of contraction or relaxation in the rectus muscles of the eyeballs.

II. The focal adjustment, interpreted through the sensation of contraction or relaxation in the ciliary muscle encircling the crystalline lens.

III. The visual angle, subtended by the diameter of the object regarded, and interpreted by recognition of the retinal area impressed but instantly and unconsciously referred to the external object.

With a view to finding, if possible, what relation these three elements bear to each other in abnormal vision like that in the stereoscope, I constructed a modification of the instrument originally devised by Wheatstone. Upon a cubical block,

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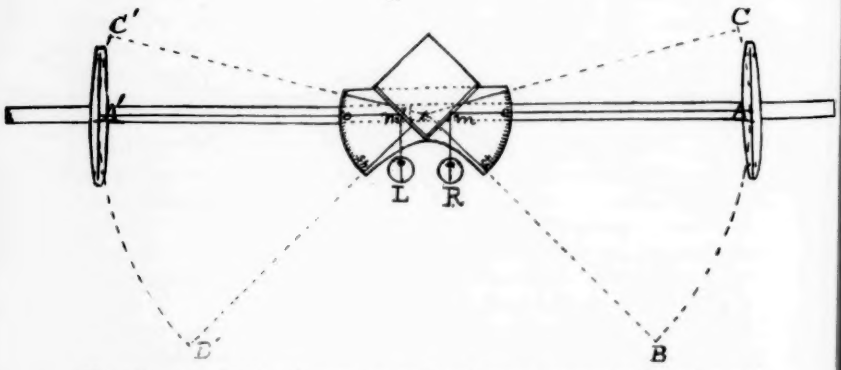

(fig. 1) two plane mirrors, *m* and *m'*, were cemented, and a pair of arms were attached to carry the conjugate pictures. A and

A′. These arms move upon a pivot, each through an arc of 60°, under a divided circle. When so adjusted that the angle of incidence on each side is 45°, the direction of the reflected rays is such as to necessitate parallelism of visual lines for those which come from the centers of A and A′ respectively. If the arms are pulled forward for example to B and B′, the angle of incidence becomes such that the eyes must be made to roll inward to retain binocular combination of images; if pushed back toward C and C′, divergence of visual lines is necessitated. The value of the optic angle, positive for convergence, negative for divergence, is obtained, with but trifling error, from the circle. On each side let the picture be kept at a fixed distance, while the eyes are as near as possible to the mirrors, for example, so that $Am + mR = 50^{cm}$. For this distance, in normal vision, the value of the optic angle would be 7° 20′. If each arm therefore be pulled forward 3° 40′, the binocular image appears in full relief about 50^{cm} in front. Shifting the head slightly to one side, the rays from one picture are reflected by a single mirror into both eyes, and the image now appears flat at the distance of 50^{cm}. The apparent distance of this flat image is obviously independent of the relation between the arms of the instrument, and serves as a standard of comparison, the optic angle, focal adjustment and visual angle all conducing to the same judgment of distance.

Modifying slightly the formula hitherto employed, we have, for the distance, D, of the optic vertex from each eye, determined by intersection of visual lines,

$$D = \tfrac{1}{2} i \operatorname{cosec} \tfrac{1}{2}\alpha,$$

where i is the interocular distance, and α the optic angle. If this equation be expressed as a curve, fig. 2, taking values of α for abscissas and values of D for ordinates, the axis of ordinates is obviously an asymptote.

Let the arms now be pulled forward until $\alpha = 37° 20'$. The corresponding value of D is 10^{cm}, while the visual angle is unchanged and the focal adjustment, if perfectly distinct vision be secured, must still be for a distance of 50^{cm}. These two elements therefore tend to counteract the suggestion due to strong convergence, and the image appears perhaps 15^{cm} or 20^{cm} distant. Its apparent diameter varies directly as the estimated distance, and is diminished to ·3 or ·4 of the original diameter. The influence of axial convergence, though partially counteracted, preponderates over that of the other elements in determining the judgment.

Let the arms now be pushed back until $\alpha = -5°$. The theoretic value of D is negative and hence physically impossible, but practically the contraction of the external rectus muscles

produces the impression of continued recession in a positive direction. The visual angle has not been changed, and the focal adjustment not enough so to produce any very perceptible decrease in distinctness of vision. The image appears perhaps 60^{cm} or 70^{cm} distant, but this estimate is quite uncertain. The apparent diameter is of course increased. The effect of constancy in the visual angle in this case seems to be the preponderating element in determining the judgment.

The results of experiment with the apparatus just described are given in the curve A A′, of Fig. 2. The stereograph em-

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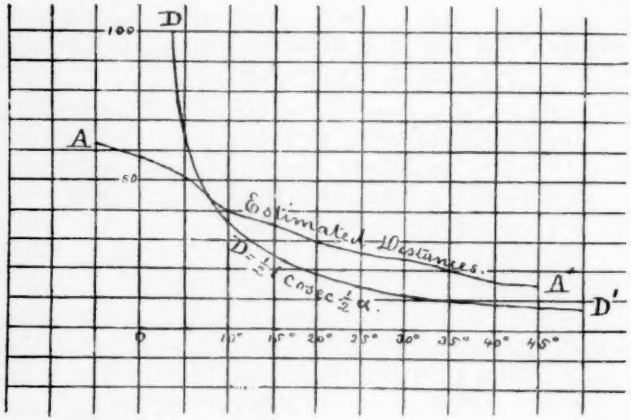

ployed was one of the full moon, so arranged as to produce reversion of relief. Distances were estimated to the edge of the concavity, which was surrounded with a uniform black surface. It is seen that the curve of theoretic distances, DD′, is cut by that of apparent distances, AA′, not far from the point corresponding to $\alpha = 7° 20'$. The stereoscope was manipulated by an assistant who varied the optic angle in irregular order, recording each value, of which I remained ignorant, and at the same time recording my corresponding estimate of distance. The curve has been constructed from the record of six independent series of estimates. In consequence of the difficulty of securing perfect dissociation between axial and focal adjustments for large positive values of the optic angle, 45° has been taken as a limit, though 80° of convergence is possible. For negative values the dissociation is not difficult, but beyond −5°, the image is apt to be unsteady. As might be expected, the curve is not regular, the probable error being ±8^{mm}. It is found almost to coincide with the theoretic curve for a short distance on each side of the intersec-

tion, but the judgment is much vitiated as we depart from the conditions of normal vision. Even for $\alpha = 7° 20'$ my estimate of distance was too small, and as a whole the curve shows strikingly how fallacious must be any conclusions drawn from Brewster's theory that there is a necessary connection between apparent distance and optic convergence, or, as he expressed it, that we "see distance," instead of judging it as contended by Berkeley.

The curve also shows that, under the conditions imposed, the variation in apparent distance is not very great between the limits of $-2°$ and $+5°$, within which the optic angle is included in most cases of binocular vision with lenticular stereoscopes. This explains my remark in a former article that the judgment of absolute distance is "in practice nearly, but not quite, independent of the optic angle,"[7] but is influenced rather by physical perspective. In these cases, it will be observed, the field of view is quite limited, and the optic angle not very large.

Sir David Brewster[8] noticed the strong effects obtained with convergence of visual lines by combining the images of perfectly similar patterns, recurring regularly and in great number, on large surfaces. When an extended field of view is occupied by such images, the effect of contraction in the rectus muscles seems to be more marked in comparison with that of the other elements of perspective, in estimating absolute distance there being no contrast of background and foreground to interfere. This enhancement is noticeable also when the visual lines are made to diverge, but still the positive visual angle is more important than the negative optic angle in determining the resultant effect. This is well shown by the following experiment: A large vertical surface is found upon which there are regularly recurring figures separated from each other by an interval 3^{mm} in excess of my interocular distance. Standing 50^{cm} off, in front, the images of contiguous pairs are combined by axial divergence, the optic angle being very nearly $-0° 21'$. The illusion is that of a papered wall about 3^{m} distant. Approaching them until the divergence is doubled, the wall appears about 2^{m} distant. The increase of divergence tends to produce the effect of apparent recession, but this is much more than counteracted by increasing the visual angle as the true distance is diminished, causing the externally projected image to appear to approach from the further side, instead of receding. The size of the retinal image varies in accordance with a well known law, but optic divergence slightly modifies the interpretation that would otherwise be suggested by the sensation. The apparent distance and diameter are thus made to vary together at will. Comparing the for-

mer, however, 3^m, with that expressed in the curve A A′, 58^{cm}, for the same negative optic angle, and the same real distance, 50^{cm}, it is seen that the change of conditions has produced a great change in the unconscious interpretation of the retinal image. In both cases the facts contradict Brewster's theory of triangulation. Brewster[3] himself noticed that when the combined image is small in comparison with the whole field of view, it did not appear at its calculated distance, even with convergence of axes, and to get rid of the disturbance due to comparison he resorted to large surfaces with geometrical patterns, but evidently without suspecting that optic divergence in viewing them was possible.

While the curve A A′, fig. 2, represents the result of experiment upon myself alone, similar results have been obtained from the examination of several other persons. In each case the range of uncertainty has been large, and the curve of theory has been found to be crossed by that of experiment near the point corresponding to the optic angle of normal binocular vision. Estimated distances were always greater or less than distances calculated by use of the formula, according as the optic angle was greater or less than that of normal vision at the true distance of the object regarded. For small and negative angles, not only is the departure of the curve of experiment from that of theory very rapid, but the uncertainty in the estimate of distance is increased. For larger angles the two curves approach more nearly to coincidence if the observer is well practiced and at the same time presbyopic, so that ciliary adjustment interferes less with the suggestion due to axial adjustment.

There are several considerations which interfere still further, in comparing results obtained from different observers. One person may be less accustomed than another to depend upon the sensation of muscular strain in interpreting visual perceptions. The definiteness of the interpretation becomes less as the departure from normal conditions increases. The same degree of convergence or divergence may imply greater muscular strain for one pair of eyes than for another, according to the elasticity and vigor of the muscles, or the age of the individual. Even when the real distance of the object is varied with the optic angle according to the formula so that there is no departure from the conditions of normal vision, the estimates of each observer will be found to be affected with a nearly constant error, as has been shown by the experiments of Helmholtz and Wundt.[4] This error was more than 33 per cent of the true distance in many of Wundt's experiments. In abnormal vision, furthermore, the dissociation between axial and focal adjustments is usually not instantaneous, and for strong

convergence it is very limited. The curve A A′ might perhaps have approached more nearly that of theory had I waited longer for ciliary adaptation, but this in turn involved great fatigue in the rectus muscles, especially when many experiments in succession were made, and the error was as great from this cause as from imperfect dissociation. Brewster observes, in reference to the binocular image obtained by forced convergence,[8] that "it generally advances slowly to its new position," and he speaks of "the influence of time over the evanescence as well as the creation of this class of phenomena." The nature of focal accommodation by action of the ciliary muscle was not then known—(1844).

3. A. New Mode of Stereoscopy.

The study of physiological perspective has enabled me to attain the explanation of a phenomenon first observed by Brewster, but not explained by him, and directly contrary to his theory of visual triangulation. In viewing with strong cross-vision a large plane surface on which are regularly recurring figures, such as wall paper, a phantom image of the wall is easily obtained, which appears suspended in mid-air. Of this image Brewster observes in passing, that "the surface seems slightly curved,"[9] but discusses this feature no further. His explanation[10] of the production of the phantom wall is easily understood. Let A, B, C, etc. (fig. 3), be equidistant points on the

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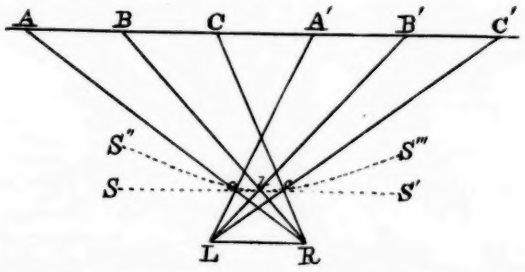

wall, in front of which stands the observer whose eyes are at R and L. If the right eye be directed to A and the left to A′, the intersection of visual lines is at *a*. In like manner, *b* and *c* are determined, and it is an obvious geometric necessity that the locus of the phantom surface must be a plane S S′ parallel to the given plane AC, assuming the line L R to be parallel to it also. It cannot be a curve S″ S‴, if the theory be true. To make the curvature apparent, the optic angle must be large: and on account of the exceeding muscular strain it in-

volves, the experiment has probably been rarely tried, and soon passed into oblivion. The curvature of the phantom surface, in a median plane passing vertically between the eyes, was rediscovered a short time ago by Professor LeConte, and soon afterward I discovered the curvature in all directions.

The effect is in no way due to intersection of visual lines, but to the opposite obliquity of vision with each eye separately, combined with the fact that the retinal surface is not plane but almost spherical at the points impressed, the center of curvature being very near the nodal point of the crystalline lens. The experiment is therefore far easier and more striking if optic parallelism or slight divergence be substituted for strong convergence, and if, instead of a wall, a pair of cards be employed, on which are perfectly similar figures, such as a pair of similar series of concentric circles. If there be difficulty in directing the eyes, an ordinary stereoscope can be used as an aid. I have devised a simple attachment for the adjustable stereoscope described in my last paper, by which one can with perfect ease thus secure stereoscopy with similar figures. Dissimilarity between the external pictures has hitherto been deemed indispensable for the attainment of true stereoscopic effects. The present method therefore, in which advantage is taken of the globular form of the eye, so far as I can learn, is entirely new. The binocular relief moreover can be reversed at will without consciously changing the relation between the visual lines, and the same pair of simi-

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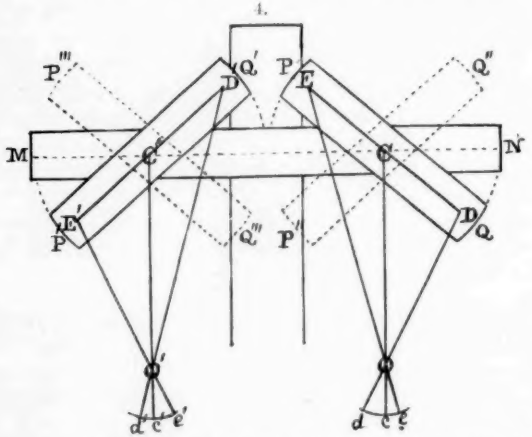

lar pictures can be examined with comfort while in form the binocular image changes from an elliptic convex shield to a flat circular plate and thence into a deep elliptic cup; the process being reversed at pleasure.

The attachment consists of an ordinary cross-bar, M N, fig. 4, which may be placed as near as convenient in front of the eyes whose optic centers are at O and O′, the visual lines being parallel and passing through points C and C′. These are the centers of the conjugate series of concentric circles, on cards whose planes are perpendicular to the principal plane of vision, and which rest on extra short bars, P Q and P′Q′. The latter are pivoted on the cross-bar so as to revolve about vertical axes passing through C and C′ respectively. Let E D and E′D′ be the horizontal diameters of the largest circles, the cards having been revolved so as to make with each other a dihedral angle opening toward the observer. Their relation to the visual lines is obviously the same as if their planes were coincident and the visual lines crossed, making the optic angle equal to the sum of N C D and M C′E′, as in Brewster's experiment. The retinal projections of E C D and E′C′D′ are ecd and $e'c'd'$. Since the triangles E O D and E′O′D′ are oblique, their medians divide the angles at O and O′ unequally; hence $dc > ce$ and $d'c' < c'e'$. The retinal images in the two eyes are hence dissimilar; and this dissimilarity may be made so great by increasing the angles N C D and M C′E′ that the binocular image becomes confused if the eyes are not made to play rapidly over the picture. If the attention be momentarily withdrawn from C and C′ to D and D′, the visual lines become divergent to an extent measured by the difference of the angles D O C and D′O′C′. The associated contraction of the external rectus muscles which this necessitates at once produces the sensation that habitually accompanies recession of the object binocularly viewed. The same is true if the attention be restored to C and C′, and then given to E and E′. The binocular image of the horizontal diameter must hence be perceived as a curve, convex toward the observer.

Each circle moreover must be projected upon the retina approximately as an ellipse whose minor axis is horizontal, its ratio to the major axis being readily calculable if the angle of inclination be known. But the retinal ellipses are no longer concentric (figures 5 and 6), the extent of retinal displacement depending on the extent of the minor axis in each. If the successive vertices be connected, we have two curved lines, A C B and A′C′B′. If these be binocularly combined and externally projected, since C C′ is less than A A′ and B B′, optic divergence becomes necessary in transferring the attention from C and C′ to A and A′ or B and B′. The binocular image of the vertical diameters must hence be perceived as a curve, convex toward the observer.

Let F and G (fig. 5) be points symmetrically situated with regard to the vertical diameter and hence equidistant from D

and E respectively. When the card is revolved, as in fig. 4, the distance O E exceeds O D, and hence the visual angle subtended by E G is less than that subtended by D F. Every ellipse therefore is distorted. To each eye separately the effect

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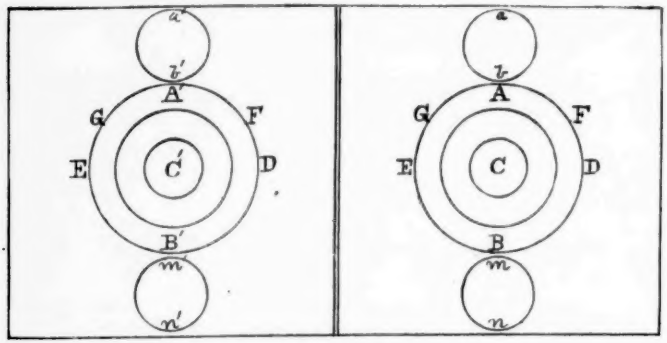

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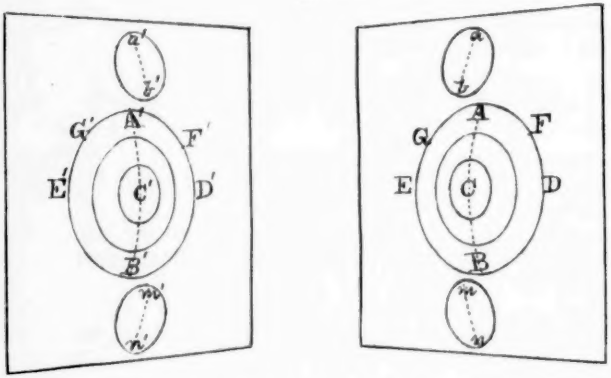

is the same as if every major axis were bent, and every point of each curve were correspondingly displaced; F and G′ are elevated, F′ and G depressed, and hence F and F′ differ in retinal latitude as well as longitude. The binocular combination, however, is perfect, although double images due to difference in retinal latitude are neither homonymous nor heteronymous. That conjugate points differing slightly in altitude can be binocularly viewed and their images combined, even when there is no horizontal stereoscopic displacement, was first shown by Professor W. B. Rogers." This is one of several considerations which show that the theory of corresponding points in binocu-

lar vision cannot be accepted in any mathematical sense. In the present case the distortions of the retinal ellipses, being equal and opposite in the two eyes, are perfectly corrected in the binocular combination of each pair; the resultant curves are hence perfect ellipses.

If a pair of small circles whose vertical diameters are $a\,b$ and $a'b'$ be drawn above the large circles, the visual lines directed to their centers are similarly oblique to their vertical diameters but oppositely oblique to their horizontal diameters. The external projections of their retinal pictures are hence slightly distorted ellipses, of which the upper vertices are farther apart and the lower vertices nearer together than their centers. The binocular combination is hence an ellipse whose plane is oblique, the upper vertex being farther, and the lower vertex nearer to the observer. A pair of small circles below the large ones are binocularly combined with opposite obliquity.

No explanation is now needed to show that if the planes of the cards be revolved into the positions P″Q″ and P‴Q‴ (fig. 4), the combination of the concentric circles must present a concave surface and the obliquity of the plane of each pair of conjugate small circles, when binocularly viewed, must be reversed in sense.

Brewster's remark about the phantom wall that "it generally advances slowly to its new position"[10] is now easily understood. When E and E′ (fig. 2) are binocularly viewed, since EO exceeds E′O′, there must be dissociation between the two focal adjustments which are generally adapted to the same distance. To this must be added the necessary dissociation between axial and focal adjustments, the former being for an infinite distance, the latter for the distance OC, when C and C′ are binocularly viewed. To untrained eyes this unusual combination of muscular actions is at first a little confusing, and for a few seconds it is often difficult to decide whether the surface appears convex or concave. Despite this inconvenience, if the experiment be performed with axial parallelism, the image soon becomes clearly defined. With strong axial convergence, as in Brewster's method, the dissociation is far more difficult on account of the extreme muscular strain that is necessary.

By holding the cards, fig. 4, with their planes coincident and then drawing them apart in this plane, so that 6° or 7° of divergence of visual lines is necessitated in retaining retinal fusion, the image changes very perceptibly from that of a flat plate to that of a shallow concavity. By cross vision the opposite is obtained; but not so strikingly, for the muscular strain of 7° of divergence I find to be as great as that of 60° or 70° of convergence.

If vertical lines, $a\,n$, $a'n'$, fig. 5, be combined binocularly

they appear as a tangent to the curved surface, and this pierces the planes of the small ellipses, passing through each at its center. Any inaccuracy in either drawing interferes with these results and is at once manifested in the binocular picture.

40 W. 40th st., New York, Feb. 25, 1882.

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[7] This Journal, Dec., 1881, p. 447.
[8] Optique Physiologique, p. 824.
[9] Brewster on the Stereoscope, p. 91.
[10] The same, p. 95.
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[12] Elementary Physiology, Macmillan & Co., p. 280.

ART. XXXII.—*On a great dyke of Foyaite or Elæolite-syenite, cutting the Hudson River Shales in Northwestern New Jersey;* by BEN. K. EMERSON.

IN the Geology of New Jersey by Professor G. H. Cook, 1868, p. 144, an extensive dyke, "on the eastern slope of the Blue Mountain," is described as follows from the manuscript notes of Mr. E. Hauesser, assistant on the earlier geological survey under Dr. Kitchell: "On the slope of the Blue Mountain between Beemersville and Libertyville, a dyke of porphyritic hypersthene rock has been intruded between the slate and conglomerate. It runs from a point one and a half miles northwest of Libertyville, without interruption, to its termination, which is one and one-fifth miles northwest of Beemersville. It is about a quarter of a mile wide, and is a little tortuous in its course, but its general bearing is northeast and southwest. The rock of the northeastern part consists usually of a coarsely granular aggregate of labradorite and hypersthene, associated occasionally with hornblende, mica, sphene, tourmaline and quartz. The labradorite occurs in imperfect crystals, and in distinctly cleavable masses; it is of a gray color and a pearly luster. The hypersthene is of a greenish black color, of a bright metallic luster; it occurs in small and slender crystals more or less perfect.

"At the southwestern extremity the dyke presents a peculiar and striking appearance. It does not occupy a long and high hill, with nearly perpendicular slopes, like the northeastern part, but, owing to a powerful and rapid disintegration, it has

crumbled into loose pieces and into a fine sand, which form a range of low hills with gentle slopes, and which, seen from a distance, look exactly like hills of sand or drift.

"The dyke here consists of a rather coarsely granular aggregate of labradorite, sphene, mica, quartz, pyroxene and iron pyrites. The sphene is of a brown to yellowish brown color, of an adamantine luster, and occurs in small, more or less perfect, crystals, in such quantity as to form one of the principal constituents of the rock.

"The iron pyrites is profusely disseminated throughout the rock, and causes the rapid decomposition.

"There is usually a marked difference between the middle of the dyke and the margins, the material being distinctly crystalline at the middle while it is compact at the surface. The dyke is traversed by joints and by some small veins, and it also sends out veins into the contiguous rocks.

"The slate has been altered for a distance of three thousand feet from the southeastern border of the dyke.

"Very near the dyke the rock is so much changed that the stratification cannot be determined, but farther off, though the rock is much changed, the marks of stratification are still plain, and the rock has the usual northwesterly dip."

Being fresh from a study of the wonderful "Cortland series," under the kind guidance of Professor Dana, my attention was immediately attracted by the description above of a great dyke of hypersthenic rock which crumbled readily into hills of coarse sand, and had exerted such powerful metamorphic influence upon the soft Hudson River rocks, and I visited the locality in the hope that it might throw some light upon the complexities of the Cortland region. Leaving the railroad at Deckertown just above Franklin Furnace, the drive of six miles was over a hilly country of soft crumbling shales. More than half a mile from the dyke its influence was manifest, the shales having changed into a dark gray compact mass, breaking into wedge-shaped pieces bounded on all sides by slickensides. Nearer at hand the shales are baked into a complete hornstone of a black color, breaking with the perfect conchoidal fracture of flint, and cut into small fragments by numerous joints. Where the road over the mountain leaves the road which skirts the mountain on the east, at the house of Mr. D. B. Roloson, this is seen in great perfection on the hillside, and in the orchard above this gentleman's house, a band of limestone occurs—a dark gray fine grained rock, which might be easily mistaken for a fine grained diorite. It has, however, the rusty corroded surface of a siliceous limestone. Scales of biotite are disseminated in it in considerable abundance, and the rock is so rich in magnetite that the coarse powder is readily

taken up by the magnet. Microscopically the rock is a crystalline-granular aggregate of calcite and magnetite.

A few rods farther south, and just behind the house in the orchard, occurs a large mass, apparently a bowlder, of a fine-grained brown-black eruptive rock, which is so filled with small angular fragments of the same siliceous limestone, that these latter make more than half its mass. They are gray, compact and horny, and contain in abundance minute cubes of pyrite arranged with some regularity parallel to the surface of the broken fragments. There also occur in the mass of the trap small fragments of quartz manifestly enclosures, and small aggregations of white grains of calcite so mixed in with scales of biotite that they seem to be the result of fusion and crystallization in the mass, rather than of subsequent decomposition. Examined under the microscope, the trap was found to be thoroughly impregnated with calcite, and, when treated with acid, effervescence appeared in every part. Besides the large scales of mica visible to the eye, more minute scales of the same red mica and grains of magnetite were scattered throughout the slide in a colorless ground, interrupted here and there by quite large square and hexagonal sections of nepheline, which gelatinized slowly under the microscope after the effervescence of the calcite had ceased. Rarely a broad rounded mass of fibrous green hornblende, decomposed nearly to the center, appeared. I was not able to discover the exact relation of this rock to the main dyke, but it was probably an offshoot therefrom, as it occurs only a few rods from the latter in the midst of the region of most active metamorphism.

Going north about a mile from the house of Mr. Roloson along the road skirting the mountain on the east to a point where the woods are cleared away and a single great oak stands by the roadside, one ascends the first slope of the ridge which rises about eighty feet like a terrace scarp, and comes out upon a broad level made up of the metamorphosed slates from which the great dyke rises at the west to form the crest of the mountain, and stretches away several miles to north and south. Toward the front of this terrace-like plain, and about thirty rods from the nearest outcrop of the dyke, a layer of the shale is altered to a tough gray rock broken by numerous fissures which has much the appearance of a claystone porphyry. In a ground mass, quite felsitic in appearance, are imbedded opaque white Carlsbad twins of orthoclase, sharply defined and six to eight millimeters in length, small rhombs of calcite and cubes of pyrite now changed mostly to limonite, and in a single instance a quite large crystal of dark green chlorite. The ground mass is resolved under the microscope into an aggregate of extremely fine scales of muscovite of irregular outline.

While the calcites are limpid, the orthoclase crystals, opaque white by reflected light, are of deep reddish brown by transmitted light, and this color, which is caused by the abundance of a very fine red dust, is spread uniformly over every portion of every crystal, except in one slide where they are quite fresh, and enclose in considerable number small perfect spheres of a deep brown to black color, which seem to be some hydrocarbon compound which has been included in the forming crystal in a liquid state. This black carbonaceous matter is scattered here and there through the mass, and it is sometimes aggregated in the midst of a brownish substance in such a manner as to suggest an incipient stage in the formation of chiastolite crystals. All the rest of the micaceous ground-mass is mottled by a great number of very minute, stout, rod-like forms, four to six times longer than broad, and quite uniform in size and distribution, though not always regular in shape, being often rounded, pointed, or ravelled out at one or both ends. They are nearly opaque but resolvable (×800) in the thinnest places into a congeries of red scales. The specimen contains a single large crystal of a dark green chlorite, visible to the eye, and these minute forms may have resulted from the decomposition of a similar mineral, but it seems to be rather an early stage in the growth of some ferruginous mineral like ottrelite.

On crossing the high terrace-like buttress of thoroughly metamorphosed rocks, here, perhaps forty rods wide, one comes upon the sharp slope of the great ridge formed by the dyke itself. The contact of the dyke and the enclosing rock was nowhere to be seen. Where it was first met with in place, a little way up the slope, the foyaite bears no resemblance to the compact trap-like rock from near Roloson's, described above. It is completely granitic in texture, of medium grain, and of a grayish flesh color, deriving this color mainly from the elæolite which is its principal constituent. The latter is so abundant that a small piece of the rock treated with hydrochloric acid changed into a thick jelly, in which only a small quantity of black ægirite remained undissolved. The hand specimens which I got from this portion of the dyke contain certainly 90 per cent of elæolite. The mineral possesses the usual greasy luster, is granular massive, yet shows under the lens everywhere incomplete crystalline outlines.

The component next in importance is ægirite, which appears in black elongated crystals up to 3^{mm} in length, upon which I was able to measure the angle of the prism 92° 47′. The mineral fuses with somewhat greater difficulty than the Norwegian ægirite, and tinges the flame yellow. The surface of many of the crystals is brightly iridescent, as is sometimes the case with the arfvedsonite from Kangerdluarsuk, and the crystals

can be seen toward the surface of the rock to be changed for a part or the whole of their length into a greenish black, fibrous, hornblendic mineral apparently arfvedsonite.

The third constituent, orthoclase, occurs in elongated Carlsbad twins up to 30^{mm} in length; in a single instance three crystals are twinned together. They are of brilliant luster, and of the same flesh color as the elæolite in the fresh rock, while toward the surface they are whitened, and there closely resemble the feldspar of the Brevig zircon-syenite. They are scattered in the rock somewhat distantly, so that in three sections no feldspar appeared. The crystals enclose rounded masses of elæolite as well as crystals of ægirite, often so thickly crowded as to give the feldspar a pegmatitic appearance.

Titanite occurs in minute crystals quite abundantly. The maximum length is about 1^{mm}. It is of bright reddish yellow color, transparent, and of adamantine luster, and is always well crystallized in both complex and simple forms, with the faces often brightly iridescent. No trace of triclinic feldspar or of hypersthene could be found.

High up along the crest of the ridge, perhaps fifty rods west of the point where the specimens were obtained which furnished the material for the above description and for microscopical study, the rock is much coarser grained, more loosely granular and decomposed, falling readily to a mass of coarse grains under the hammer.

The proportion of elæolite is here much less, and the only additional mineral found in this coarser variety was a glossy black mica, which gives all the blowpipe reactions of astrophyllite, but it may be a biotite containing much manganese.

Under the microscope by far the greater portion of the section is elæolite, with scattered crystals of ægirite and titanite. The elæolite presents itself in three forms, quite distinctly demarked: (*a*) as distinct crystals, stout hexagonal prisms with end faces, of much the same appearance and properties as in the more modern nepheline rocks. These crystals are rare. The cleavage is more perfect and delicate than in the remainder of the elæolite in which they are embedded, and as a result they are only slightly decomposed. They are quite free from enclosures. The second form (*b*) appears in somewhat larger, aggregated, and more imperfect crystals, in which the basal, prismatic and pyramidal cleavages are ruder and more open, and the crystals much more decomposed. Flakes of a green hornblendic mineral, probably arfvedsonite, appear as enclosures parallel to the three cleavages. Finally it occurs (*c*) filling up the interstices of the other two forms like the quartz of a granite, and swarming with minute acicular microlites, especially in the recesses made by neighboring crystals of earlier formation. This portion has a less distinct cleavage and is less decomposed.

The freshest ægirite shows six-sided cross-sections (*I*, *i-i*,) remarkably regular both in form and cleavage, the latter being in straight equidistant lines carried generally clear across the crystal, and thus differing widely from that of augite. It is red brown with much depth of color in cross-sections, blackish brown in longitudinal plates, free from enclosures, and without trace of fibrous structure. Both in lengthwise and cross sections it absorbs the light with a single Nicol's so strongly that at the point of extinction it becomes jet black. The pleochroism is also very marked, ranging from charcoal brown to deep emerald green. In other crystals, starting from one end, the color in ordinary light changes gradually from dark brown into a bright green, and corresponding with this change of color the mineral becomes quite suddenly fibrous, and filled with minute grains of magnetite, which are wanting in the unchanged portion. The changed portion has all the microscopic peculiarities of a fibrous hornblende. In longitudinal sections of the ægirite a cleavage parallel to *O* appears, and in one instance occurred a well terminated arrow-headed twin—twinning plane *i-i*. Its crystals often radiate from or enclose a crystal of titanite.

The titanite appears in wine-yellow, limpid, much fissured, cross-sections, distinctly pleochroic, rough of surface, never sinking to the smallest dimensions, or containing any enclosures. Besides ægirite the microlites often radiate from it. Beautifully distinct twins occur with the twinning plane *O*, and with crossed Nicols the light is extinguished at 33° on either side of this plane.

Sodalite occurs rarely in wholly apolar patches irregularly bounded by the other constituents, and showing rude dodecahedral cleavage.

The order of crystallization of these minerals is interesting. The titanite is plainly the first formed, and had reached its limit before the appearance of the other constituents, except perhaps the first form (*a*) of the elæolite. These are almost without enclosures of any kind. Next the second portion of the elæolite (*b*) crystallized out, and during the time of its formation the hornblendic mineral, which is included in it in scales, as well as the ægirite which radiated from the earlier formed titanite, appeared. This green hornblendic mineral as it is synchronous with the ægirite and as it is found in the most decomposed portion of the elæolite, I take to be arfvedsonite paramorph after the ægirite.

Toward the close of this period the orthoclase, which encloses the imperfect crystals of the elæolite and the ægirite, separated out, and all these minerals share the same freedom from microlites, a great swarm of which is included in the third form of

the elæolite (*c*) which has taken its shape entirely from the preceding minerals and which thus closed the series.

Out on the plateau of metamorphic rock I picked up a curious mass not in place, which showed a contact of foyaite on foyaite of different age. The older was coarser grained than I found it elsewhere and richer in orthoclase, being largely made up of crystals 25–30^{mm} long, distributed porphyritically in a fine-grained rusty ground containing pyrite. The newer rock, resting on this, has a rudely columnar structure at right angles to the plane of contact, resembling somewhat the gypsum crusts from salt vats.

The columns of which this layer consists are elongate imperfect crystals of orthoclase and elæolite and bundles slightly radiated of a greenish black hornblendic mineral. Small tufts and spheres of the latter are also scattered through the mass, together with some dark purple fluor and much pyrite.

Under the microscope distinct crystals of elæolite appeared and the hornblende needles were contracted, like the handle of a dumb-bell as they passed through such a crystal, showing that the hornblende had here commenced its crystallization first and that the two minerals had thereafter increased together.

The hornblende crystals are now in an advanced stage of decomposition, being resolvable into an aggregate of short interrupted fibers and so impregnated with scales of "ferrite" as to be often quite opaque, and the same material has also often gathered on the needles in large masses like the warts on a plum twig. Only rarely in unchanged non-fibrous portions of green color could the angle of extinction of hornblende be measured. The feldspar was orthoclase with no indication of the presence of microcline.

The aphanitic and the granitic varieties of the rock described above are connected by intermediate grades of fine grain in which occur segregations in size and shape like a hen's egg, composed of coarse granular orthoclase, ægirite, pyrite and fluor.

Decomposition has everywhere affected the superficial portions of the dyke, and at the summit where the rock is coarsest, extended quite deeply, but without the formation of any zeolitic minerals.

Amherst College, Massachusetts, April, 1882.

ART. XXXIII.—*Notice of the remarkable Marine Fauna occupying the outer banks off the Southern Coast of New England, No. 5*; by A. E. VERRILL. (Brief Contributions to Zoology from the Museum of Yale College: No. LI.)

ANTHOZOA (*continued.*)

Pennatulacea.

PREVIOUS to 1871 no representative of the Pennatulacea had been discovered on the American coast between Cape Hatteras and the Arctic Ocean. In that year, and also in 1872, a number of specimens of *Pennatula aculeata* and of a small *Virgularia* were dredged by Mr. J. F. Whiteaves in the deeper parts of the Gulf of St. Lawrence. In 1872 these were also dredged by Dr. A. S. Packard and Mr. C. Cooke, on the "Bache," in the Gulf of Maine (see this Journ., v, pp. 5, 100, 1873). Subsequently the former species has been obtained in many localities, and in large numbers, off the coasts of New England and Nova Scotia.

Since the summer of 1878, when the Gloucester fishermen became interested in bringing home, for the use of the U. S. Fish Commission, the various objects taken on their deep-sea lines, they have not only presented large numbers of this species, from many localities, but they have contributed several others that are of still greater interest, including more than a hundred fine specimens of the very large *Pennatula borealis*, many of them nearly two feet long, with about as many of *Ballicina Finmarchica* and *Virgularia* (*Anthoptilum*) *grandiflora* V., together with a few specimens of *Funiculina armata* V. and a *Halipteris*. All these interesting additions to our fauna have already been recorded by me, in 1878 and 1879, in this Journal, in former articles of this series. All these species have also been dredged by our parties on the "Fish Hawk."

Nevertheless, in the recent work of Professor Kölliker on the Pennatulacea of the Challenger expedition,* in discussing the distribution of the group, he states that no Pennatulidæ are known on the Atlantic coast of America. This was the case eleven years ago, but the reverse has been true since that time. I have, at the present time, more than a thousand examples of our two species of *Pennatula* in my possession.

All the species will be fully described and figured, and the details of their distribution will be given, in a report on our Anthozoa, soon to be published in the report of the U. S. Fish Commission.

* The Zoology of the Voyage of H. M. S. Challenger. vol. i, Part ii, Report on the Pennatulida, by Professor Albert V. Kölliker, 1880.

Pennatula aculeata Danielssen and Koren.

Pennatula aculeata Danielssen, Forhandl. Vidensk.-Selsk. Christiania. 1858, p. 25; Fauna Littoralis Norvegiæ, iii, p. 86, pl. 11, figs. 8–9. 1877.
Verrill, this Journal, v. pp. 5, 100. 1873.
Pennatula phosphorea, var. *aculeata* Sars; Kölliker, Alcyonarien, i, Pennatuliden, p. 134, pl. 9, fig. 73. 1870.

This species is very abundant and widely distributed on our coasts, in 100 to 487 fathoms, on soft muddy bottoms. Gulf of St. Lawrence, 160–200 fathoms,—Whiteaves, 1871–3; Gulf of Maine,—U. S. Fish Commission, on the "Bache," 1872–3; Grand Bank, St. Peter's Bank, Banquereau, Western Bank and other banks off Nova Scotia, in 60 to 300 fathoms,—Gloucester fishermen (in 29 lots, including about 90 specimens); off Cape Sable, N. S., 88 fathoms,—U. S. Fish Commission; off Martha's Vineyard and Block Island, and off Chesapeake and Delaware Bays, 1880, 1881, in 100–487 fathoms,—U. S Fish Commission. Several hundreds of specimens were taken at each of the stations 943, 945, 1025. Also taken by Mr. A. Agassiz, on the "Blake," 1880, in 197 to 524 fathoms. Chistiansund, 30–100 fathoms,—Sars and Danielssen. Eastern Atlantic, 300 fathoms,—Carpenter and Thomson.

Variety, *rosea* Danielssen, op. cit., p. 88.

Several fine specimens of this handsome variety occurred among a large number of the usual dark red variety, at stations 943, 945, 1028. It differs only in color.

Variety, *alba* Verrill.

This name is used to designate a pure white variety, which was taken at station 1025, in 216 fathoms.

Pennatula (*Ptilella*) *borealis* Sars, sp.

Pennatula grandis Ehrenberg, Corall. rothen Meeres, p. 66, 1832, (*non* Pallas).
Kölliker, Zool. Voy. Challenger, i, pt. ii, p. 4, 1881.
Pennatula borealis Sars, Fauna Lit. Norvegiæ, i, p. 17, pl. 2, figs. 1–4, 1856.
Kölliker, Pennatuliden, i, p. 136.
Verrill, this Journal, xvi, p. 375, 1878.
Ptilella borealis Gray, Catalogue of Sea Pens, p. 21.
Verrill, this Journal, xvii, p. 241, 1879.
Ptilella grandis Koren and Danielssen, Fauna Lit. Norvegiæ, p. 82, pl. 11, figs. 1–7, 1877.

One young specimen of this magnificent species was dredged by us at station 925, in 224 fathoms. From the Gloucester fishermen over 120 specimens, mostly of large size, have been received by the U. S. Fish Commission, all of which have been examined by me. These were received in 83 lots, from 1878 to 1881. They were taken in 120 to 350 fathoms, on the outer slopes of the Grand Bank, St. Peter's Bank, Western Bank, Banquereau, Sable Island Bank, Le Have Bank and George's

Bank. Previously it was known from only a few Norwegian specimens, from Chistiansund, Bergensfjord, Lofoten, Banenfjord, etc., in 150 to 200 fathoms.

Balticina Finmarchica (Sars) Gray.

Virgularia finmarchica M. Sars, Fauna Lit. Norvegiæ, ii, p. 68, pl. 11.
Balticina finmarchica Gray, Catalogue of Sea Pens, p. 13.
Verrill, this Journal, xvi, p. 375, 1878.
Stylatula finmarchica Richiardi, Monografia della Fam. Pennatularii, p. 69, 1869.
Pavonaria finmarchica Kölliker, Pennatuliden, p. 243, 1871 (*non Pavonaria* Cuvier).

Several specimens were trawled by us, off Martha's Vineyard, in 160 to 238 fathoms, in 1880 and 1881. The Gloucester fishermen have presented many large and fine specimens (more than 75), some of them over two feet long. These came in 57 lots, from the outer slopes of the Grand Bank and all the banks off the Nova Scotia coast, in 60 to 400 fathoms. Off George's Bank, 980 fathoms,—A. Agassiz, 1880. It was previously known off Finmark, 240 fathoms; Bergenfjord, 300 fathoms.

While a majority of these specimens are large and perfect, some examples of this, and other related species, have more or less of the distal portion of the axis bare, and sometimes bare or partially bare portions of the axis are seen along the middle region; not rarely, as much as one-half of the whole length is bare. This, I am convinced, is entirely due to accidental injuries received while still living.* In most cases the bare portions of the axis, whether terminal or median, have one or two, and often several, actinians firmly attached to them. The most common of these is a verrucose *Urticina* (probably the young of *U. longicornis* V.), which closely clasps the axis with its basal membrane, the edges of which unite firmly together by a suture, where they meet, thus surrounding the axis like a sheath. But other species of actinians also occur upon them, in the same way, and also hydroids, bryozoa, sponges, etc.

The very young specimens of this species have the polyps isolated, thus forming two simple alternate rows, along the rachis. These agree essentially with the genus *Microptilum* Kölliker (Challenger Voyage, Pennatulida, page 26). Indeed, I am convinced, after examining the young of several genera of Virgularidæ, that the several genera proposed by Kölliker for the small forms, with polyps in two simple rows, and for which he has proposed the families Protocaulidæ and Protopti-

* Koren and Danielssen mention the same peculiarity as occurring in all their specimens of *Virgularia affinis* and other European species. But they seem to think that this is a normal feature. They correctly object to the view that it is due to contraction in alcohol. (Fauna Lit. Norvegiæ, iii, p. 91).

lidæ (op. cit., p. 26), are mostly, if not all, the young of larger and more complex forms.*

Anthoptilum grandiflorum Verrill.

Virgularia grandiflora Verrill, this Journal, xvii, p. 239, March, 1879.
Anthoptilum Thomsoni Kölliker, Zool. Voy. Challenger, Pennatulida, p. 13, pl. 5, figs. 16–18, 1881.

The new genus, *Anthoptilum*, has been constituted, for species allied to this, by Professor Kölliker, (Voyage of the Challenger, Pennatulida, p. 13, 1881). He described, from a single specimen, a species (*A. Murrayi*, pl. 6, figs. 19–21) taken off Halifax, in 1250 fathoms, which is smaller and more slender than my species, with fewer polyps and larger zooids, but it may possibly prove to be the young form. From off Buenos Ayres, in 600 fathoms, he described *A. Thomsoni*, which is a large species, apparently identical, in all respects, with my species, from off Nova Scotia and New England.

We trawled this species, off Martha's Vineyard, in 302 to 310 fathoms. Its color, in life, is usually deep salmon-brown, but varies to pale salmon, and even to yellowish white. The Gloucester fishermen have presented about forty specimens, in twenty lots. These are from near the Grand Bank, St. Peter's Bank, Western Bank, Banquereau, Sable I. Bank, and Le Have Bank, in 85 to 300 fathoms. Off C. Fear, S. C., 647 fathoms.—A. Agassiz.

Funiculina armata Verrill.

This Journal, vol. xvii, p. 240, March, 1879.

Two excellent specimens of this fine species were taken at Stations 880 and 881, in 252 and 325 fathoms. These were larger and more developed than the original specimen, which was taken off Sable I., N. S., in 300 to 400 fathoms.

In our largest examples the polyps are arranged in oblique rows of two to four, and the zooids, which are comparatively large, are scattered along the median ventral surface, and between the transverse rows of polyps. The polyps are deep purple; the zooids are pale yellowish, with stripes of purple.

* The genus *Protoptilum* Köll., I regard as the young of *Virgularia*. His *P. aberrans* and two other similar forms, taken off New York, in 1240 to 1700 fathoms, are apparently the same as some that we have dredged in shallower water, but among our specimens, some of the larger ones have the polyps in distinct groups of two or three; these are evidently the young of a *Virgularia*. We may, for the present, designate this form as *V. aberrans*, but it may be identical with some previously known form. The genus *Trichoptilum* Kölliker (op. cit., p. 29), I consider the young of *Funiculina*. The only species, *T. brunneum*, from off Ceram, very closely resembles the young of my *F. armata*, and I should, therefore, prefer to name it *Funiculina brunnea*. *Protocaulon* may, likewise, prove to be the young of *Anthoptilum*.

MADREPORARIA.

Flabellum Goodei Verrill.

This Journal, xvi, p. 377, 1878.

This species is very closely allied to *F. alabastrum* Moseley,* taken by the Challenger, off the Azores, in 1000 fathoms. The two forms may eventually prove to be identical, when directly compared, but none of the numerous specimens examined by me agree precisely with those described and figured by Moseley. His specimens have the calicles more oblong, with the ends obtuse, instead of acute, and the costæ are much larger, more prominent and angular; and the whole surface is rougher.

This very fragile coral has the power of restoring itself from mere fragments. Many pieces have been found showing new calicles, in all stages of progress, arising from the inner surface of the fragments of old calicles. Specimens thus restored are often irregular in form till of considerable size, and one, about an inch across, still had a nearly circular outline.

In life the color of the disk and tentacles is rich salmon; lips darker salmon, with stripes of dark purplish brown, or sometimes uniform madder-brown. The larger tentacles are stout, tapered, subacute. Our largest examples are from off Chesapeake Bay, in 300 fathoms; these are 80^{mm} high; greater diameter, 120^{mm}; lesser, 43^{mm}. A specimen from station 1029, 458 fathoms, is 65^{mm} high, greater diameter 98^{mm}, lesser diameter, 50^{mm}.

The Gloucester fishermen have presented it in eleven lots, mostly from near the Grand Bank, Banquereau, Sable Island Bank, and east of George's Bank, in 180 to 400 fathoms. We have dredged it, on the Fish Hawk, at various localities off Martha's Vineyard and Nantucket, in 219 to 487 fathoms; off Chesapeake Bay.—Captain Tanner. Large numbers were taken at stations 893–895, 925, 951, 952, but they were mostly crushed to small fragments by the great quantities of larger animals in the trawl.

Bathyactis symmetrica Moseley.

Fungia symmetrica Pourtales, Deep Sea Corals, p. 46, pl. 7, figs. 5, 6.
Bathyactis symmetrica Moseley, Zool. Voy. Challenger, part vii, p. 186, pl. 11, figs. 1–13, 1*a*–13*a*, 1881.
? *Fungiacyathus fragilis* M. Sars, in S. O. Sars, Remarkable Forms of Animal Life, i, p. 58, pl. 5, figs. 24–32, 1872.

Numerous broken specimens of this very fragile coral were dredged at stations 879, 880, 895, in 225–252 fathoms, 1880. Some of these specimens must have been at least 20^{mm} in

* Proc. Royal Soc., 1876, p. 555; and Zoology Voyage Challenger, Part vii, Rep. on Corals, p. 169, pl. 7, figs. 1–2*b*, pl. 16, fig. 11, 1881.

diameter. They agree in all respects with the larger specimens figured by Moseley.

This coral is remarkable for having a wider range in depth and geographically than any other known species. It was taken by Pourtales, off Florida. By the Challenger it was taken in the N. Atlantic, off the Azores and off Bermuda, in 32 to 1075 fathoms; in the S. Atlantic, in 1900 to 2650 fathoms; in the South Indian Ocean, in 1600 to 1950 fathoms; in the Malay Archipelago and West Pacific, in 360 to 2440 fathoms; east of Japan, in 2300 to 2900 fathoms; off Valparaiso, in 1375 fathoms.

The *Fungiacyathus fragilis* of Sars closely resembles this coral, but its septa are not united into groups, nor are there any transverse dissepiments nor trabiculæ in the four specimens described, although some of them were larger than many of the specimens of *Bathyactis*, in which these characters are well marked. A larger series of the arctic form may, however, serve to unite them.

ACTINARIA.

Adamsia sociabilis Verrill, this vol., p. 225.

This starts upon a small shell, usually a pteropod (*Cavolina*), occupied by the crab, but eventually secretes a chitinous pelicle and absorbs the shell. The base becomes much expanded and bilobed, the lobes often surrounding the aperture of the shell, and uniting. Column slender and long in full expansion, very changeable, smooth, with pores near the base; disk a little wider; tentacles small, slender, in two circles, alternately erect and recurved. Mouth often protruded.

Sagartia abyssicola Verrill.

? *Phellia abyssicola* Koren and Dan., Fauna Lit., Norvegiæ, iii, p. 78, pl. 9, figs. 3, 4, 1877.

Base broad, adherent, expanded and thin at the edges, often clasping the tubes of *Hyalinœcia*, stems of *Acanella*, etc. Column narrow in the middle, expanded at summit, usually covered, except near summit, with a closely adherent coating of sand, foraminifera, etc., but many specimens are entirely naked. Tentacles rather slender, acute, forming about three marginal circles, more than half as long as the diameter of disk. Column flesh-color; tentacles sometimes pale flesh-color, sometimes salmon, frequently dark purplish brown, in the paler examples often with a darker streak on each side of the base; flake-white patches often occur between their bases, or at the margin; disk usually orange-brown or purplish brown, with lighter and darker radii. Pink acontia are often emitted abundantly from pores scattered on the column, and from the mouth.

Very abundant in this region, in many localities, on pebbles, shells, worm-tubes, *Acanella*, etc.; 76–506 fathoms.

List of Anthozoa.

PENNATULA ACULEATA Kor. & Dan. 100 to 487 fathoms.

S. 869, 873, 875, 878 ab., 880, 892, 895 ab.: 897: 924, 925, 938, 943 ab., 945 very ab., 946 ab., 951, 999, 1025 very ab., 1026 ab., 1028, 1029, 1032, 1045.

PENNATULA ACULEATA, var. ROSEA Kor. & Dan. S. 943, 945, 1028.

PENNATULA ACULEATA, var. ALBA Verrill. 216 fathoms. S. 1025.

PENNATULA (PTILELLA) BOREALIS Sars. 224 fathoms. S. 925 1 j.

BALTICINA FINMARCHICA Gray. 160 to 238 fathoms.

S. 895: 924, 925, 945, 951.

VIRGULARIA, sp. (young). 487 fathoms. S. 892.

ANTHOPTILUM GRANDIFLORUM V. [=VIRGULARIA GRANDIFLORA V.] 302 to 310 fath. S. 938, 998.

FUNICULINA ARMATA Verrill. 252 to 325 fathoms. S. 880, 881.

ACANELLA NORMANI Verrill. 219 to 458 fathoms.

S. 880, 881 ab., 893 ab., 894, 895: 937, 938 very ab., 947 very ab., 951, 1028, 1029 very ab., 1031.

ANTHOTHELA GRANDIFLORA (Sars) V. 255 fathoms. S. 1031.

ANTHOMASTUS GRANDIFLORUS Verrill. 410–458 fathoms. S. 1028, 1j., 1029 ab.

ALCYONIUM CARNEUM L. Agassiz. 8 to 30 fathoms.

ADAMSIA SOCIABILIS Verrill, sp. nov. 86 to 410 fathoms.

S. 869, 870–873, 875–878: 898: 923, 940–941 very ab., 1027 ab., 1028, 1035 ab., 1036 ab.: 1043.

SAGARTIA ABYSSICOLA (Kor. & Dan.) V. 76 to 506 fathoms.

S. 869, 873, 880, 892, 895, 894: 897: 923–925 ab., 937–939, 940 ab., 941, 949, 951, 1025, 1028, 1032 ab., 1033, 1038 ab., 1039: 1043, 1047.

URTICINA (TEALIA) CRASSICORNIS Ehr. 16 to 40 fathoms.

URTICINA CRASSICORNIS Ehr. (?) Young. 86 to 146 fathoms.

S. 872 ab.: 923, 949 ab., 1038 ab., 1039, 1043.

URTICINA (TEALIA) LONGICORNIS Verrill. 120 to 325 fathoms.

S. 869, 876, 877, 879–881: 924, 938, 945, 1032, 1033, 1043.

URTICINA LONGICORNIS V. (?) Young.

S. 924, 925, 945, 951. On *Balticina*.

URTICINA (TEALIA) PERDIX Verrill, sp. nov. 61 to 115 fathoms.

S. 871, 872: 920, sev. l., 921, 922.

URTICINA (TEALIA) NODOSA (Fabr.) Verrill. 100 to 506 fathoms.

S. 869, 872, 876–881, 893–895, 898: 924 ab., 925, 937–939 ab. l., 945, 949, 951, 997, 998 ab., 1025, 1026, 1028, 1029, 1031–1033, 1038, l.

URTICINA CALLOSA Verrill, sp. nov. 120 to 335 fathoms.

S. 876, 879–880 ab. l., 894: 924, 946, 951, 997, 998, 1025, 1031, 1033.

URTICINA CONSORS Verrill, sp. nov. 160 to 458 fathoms.

S. 924, 938, 939 (3), 947, 1029.

ACTINERNUS SAGINATUS Verrill, sp. nov. 458 fathoms. S. 1029.

BOLOCERA TUEDIÆ Gosse. 65 to 365 fathoms.

S. 879–880 ab., 894, 895: 921, 924 ab., 925 l., 938–939 ab. l., 1031–1033.

CERIANTHUS BOREALIS Verrill. 100 to 258 fathoms.

S. 873, 875, 876, 878, 897: 939.

EDWARDSIA FARINACEA Verrill. S. 1038.

EPIZOANTHUS AMERICANUS Verrill. 28 to 487 fathoms.

S. 865, 869–878 very ab., 880, 892, 894, 895, 896: 898 ab., 899: 918–924 very ab., 939, 940 ab., 941, 944 very ab., 945 ab., 949 ab., 985–990, 997–999, 1025, 1027, 1032 ab., 1035 ab., 1036 very ab., 1038, 1039, 1040, 1043, 1046.

EPIZOANTHUS AMERICANUS V., encrusting variety (=*Zoanthus Norvegicus* Koren & Dan.) 69–160 fathoms.

S. 871–873 ab.: 922–924, 940 ab., 941, 949, 1036, 1038 ab., 1039, 1040, 1043, 1046.

EPIZOANTHUS PAGURIPHILA Verrill, sp. nov. 252 to 458 fathoms.

S. 880, 883, 893, 894: 938, 947 very ab., 994, 997, 998, 1028, 1029.

PARACYATHUS, sp. 65 fathoms, on shells. S. 865.

FLABELLUM GOODEI Verrill. 219 to 487 fathoms.

S. 879, 880, 892, 893–895 ab., 898, 3 l.: 925 ab., 938, 946, 951 ab., 952 ab., 1028, 1029, 1031.

BATHYACTIS SYMMETRICA Moseley. 225 to 252 fathoms. S. 879, 880, 895.

PARASMILIA LYMANI Pourt. 57 to 130 fathoms. S. 899 ab.: 940 ab., 949, 1040.

ART. XXXIV.—*On the Determination of Phosphorus in Iron;* by J. LAWRENCE SMITH, Louisville, Ky.

IN recent years it has been a matter of considerable interest to determine the amount of phosphorus in the iron used in the arts, especially in that form of it known as pig iron; in fact, since the manufacture of steel by the process of conversion known as the Bessemer process, it has become a necessary procedure to ascertain the peculiar fitness of the cast-iron for this purpose; and has a bearing also upon the commercial value of pig-iron. It is not many years ago that a few thousandths in the difference in the amount of phosphorus in two lots of pig-iron had but little effect upon its commercial value, while now, its presence affects it to the extent of several dollars value per ton.

Formerly but little reliance was to be placed upon the analytical estimate of the amount of these small percentages of phosphorus in iron, and uniform results were not furnished by different analysts. It was not until the use of an acid solution of molybdate of ammonia* was employed, that reliable results were to be had. Many years previous (in 1852), when I established the constant presence of phosphorus in meteoric iron, I was obliged to devise a process which, while it gave very good results, was not applicable to the present use.

The so-called molybdic acid process did not at first satisfy

* Method of making described in Fresenius's Analytical Chemistry.

all chemists, and in hands of all did not give the uniform results desired, and even at the present time, modifications are constantly being sought after.

The molybdic acid process, as first used, was to precipitate the phosphorus from a nitric acid solution of the iron by an acid solution of the molybdate of ammonia, redissolve the molybdate of ammonia in ammonia, and precipitate the phosphorus by chloride of magnesia and ammonia, and estimate the amount of phosphorus from the phosphate of magnesia.

The reasons for not estimating the phosphorus by the first molybdate precipitate were, first, the supposed indefinite nature of the precipitate, and secondly, the occasional presence of free molybdic acid in the precipitate; it was also recognized that the large amount of iron in the solution interfered materially with the precipitation of all the phosphorus, and it was made very apparent that the phosphorus must be concentrated into a small portion of the iron before commencing the process of analysis.

With these facts well established to my mind, I have been engaged off and on for two or three years examining the question of the determination of phosphorus in iron and steel, making several hundreds of variously modified experiments, and repeating the details of processes adopted by different chemists.

I first tried the solution of from one to three grams of iron or steel in aqua regia, and precipitating by the molybdic solution, with all the iron present and without separating the silica; but the process gave no satisfaction, whatever way the phosphorus was ultimately weighed; nor did the evaporation to dryness over a water-bath and redissolving with a little nitric acid materially improve it. It became very evident, as already recognized by several English and American chemists, that silica must not be in the solution in which the molybdic precipitate was made; and, furthermore, that the larger portion of iron must be eliminated from the solution before this precipitation was attempted. The chemist of the Burdon Iron Works, Troy, and of the Pennsylvania Central Railroad gave me their experience on the subject.

In describing the following method, ultimately adopted as affording the most speedy and accurate results, I give but little else than slight modifications of methods already employed by others, with such detail of manipulation as facilitates uniform method of operation.*

Quantity of iron employed.—It is customary to employ 1 gram for pig-iron, and 2 to 3 grams for malleable iron and steel; but in my own practice I employ but 1 gram for

* S. Peters has used a process nearly the same as the Burdon Iron Works, Troy.

all varieties of iron; for even where the iron or steel contains one-thousandth and less of phosphorus, I get as satisfactory results as where 2 and 3 grams are employed.

Solution.—The iron, say 1 gram, is placed in a porcelain capsule of about from 100 to 150 c. m., and 3 or 4 c. m. of water added; the capsule is placed on a water-bath, and 10 to 15 c. m. of aqua regia is added little by little; the aqua regia is prepared in advance in the usual way with two parts chlorhydric acid and one part nitric acid. The contents of the capsule are now evaporated to dryness over the water bath or more speedily on an iron plate; the capsule with its contents is then placed in an air-bath and heated from 140° to 150° C. for from 30 minutes to 1 hour—thus rendering all the silica insoluble; 3 or 4 c. m. of chlorhydric acid with an equal quantity of water are added to the dry residue, and then warmed gently over a water bath or lamp; the iron is redissolved, a little more water added, the solution filtered with the filter pump; the filtrate placed on a narrow graduated measure of 100 c. m. capacity and sufficient water added to make the liquid contents 100 c. m.; the whole is well shaken to make the solution uniform. The next step is to concentrate all the phosphorus into a limited amount of the iron.

Concentration of the phosphorus.—From 90 to 92 c. m. of the last solution is placed in a capsule of 300 or 400 c. m. capacity, either of porcelain or platinum—the latter I use by preference—and 100 c. m. of water added; the iron oxide is now reduced to iron protoxide by soda sulphite or ammonia sulphite.* I prefer the latter, and prepare it in the manner mentioned in the note; the ammonia sulphite I used at the suggestion of Mr. S. Peters, which he stated to me was used advantageously by himself and others. Two or three centimeters of the ammonia sulphite is added to the iron solution and the contents of the capsule are *boiled until all the sulphurous acid is driven off*, this stage of the process being recognized by the sense of smell. By putting a small drop of the solution on the end of a glass stirrer into a weak ammonia solution we readily recognize the complete conversion of the oxide, for the precipitate is nearly white. Of course during the whole of the above process the solution is acid, with the excess of chlorhydric acid. Ammonia is now added slowly to the warm solution until a little of the greenish precipitate remains undissolved; about 20 c. m. of acetic acid is now added to the solution (which immediately redissolves the precipitate), and then

* Equal parts of ammonia and water are placed in a bottle and an excess of sulphuric acid passed through; the operation lasts for several hours, using a mixture of charcoal and sulphuric acid. Once prepared it keeps very well, when kept from the light.

1 or 2 c. m. of ammonia acetate solution; finally, the 8 or 10 c. m. of original solution remaining in the graduated glass is added with 200 or 300 c. m. of water.

The whole contents of the large capsule is boiled gently from one-half to one hour, and if necessary the water renewed as it is evaporated. The result is the formation of a basic persalt of iron containing practically all the phosphorus that was originally in the gram of iron used.

Separation of the phosphorus from the above precipitate.—With a filter-pump on a 3½-inch filter, the last precipitate is collected in 15 or 20 minutes; the precipitate is not washed, but a mixture of 5 or 6 c. m. of chlorhydric acid, with an equal quantity of water, is warmed in the capsule in which the boiling has taken place, so as to dissolve the adhering oxide of iron; the hot acid solution is thrown on the filter in the funnel, detached from the pump, the filtrate is readily dissolved and passes in some convenient vessel, and the filter washed once or twice; this solution is placed in a porcelain capsule and evaporated to dryness over a water-bath or on a hot plate. I prefer the former, although it takes a longer time. To the dry, but not over-heated residue is added 1 to 2 c. m. of nitric acid, with an equal quantity of water; this will furnish a clear solution if there be no titanium in the iron; if the latter be present, there will be formed a flocculent precipitate that can be readily separated by a filter prior to the last treatment.

The last treatment.—The solution now need not be more than 10 or 20 c. m., to which ammonia is to be added until the precipitate first formed is no longer redissolved; then add a few drops of nitric acid to clear up the solution completely, in which the phosphorus is supposed to have been concentrated. 30 c. m. of molybdic acid solution is now added to the last solution in a small beaker which is then warmed for 15 or 20 minutes to a temperature of 80° C., and agitated with a glass rod. The phosphorus is precipitated as the double ammonia-salt, and settles as a chrome-yellow powder in less than 30 minutes, and is ready for collection on a double filter,* although I commonly allow two hours or more time to elapse before filtering and washing with the filter-pump. As the filter is very small it is readily washed with a little distilled water.

* Whenever I filter a participate to be weighed on the filter, a double filter is used, each of the same size; they are weighed one against the other and exactly balanced by the weights; on the lighter one a + mark is put with pencil, and the number of mg. that it is lighter than the other. As only a 2¼ or 3-inch filter is used, the difference in weight between the filters does not usually exceed 10 or 20 mg. I always keep a number of these double filters (with the difference marked on them) ready for this purpose or any other.

After washing, the double filter is placed in an air-bath heated to about 120° C., and in about 30 minutes weighted by separating the filters, the complete dryness is verified by a second heating in the air bath.

Of the phospho-molybdate every 100 m. g. will contain 1·63 m. g. of phosphorus, or 3·74 m. g. of phosphoric acid. The result of this method of analysis will indicate a very minute quantity of phosphorus less than what is contained in the iron, but so small as not to affect the practical result, and will be more accurate, certain and speedy than if estimated as magnesian phosphate.

Cold short iron.—It has been customary to attribute the cold shortness of certain iron to the presence of phosphorus. Now, after working on this problem in rolling mills, I have found that the phosphorus cannot alone account for this peculiarity. Very often I have taken a 1-inch and 1½-inch iron that was very cold short and working them down to smaller sizes, as ½-inch bars, etc., found that very good merchantable iron is produced, capable of being bent and forged cold or hot as well as any good quality of iron, although the phosphorus in the large and small iron is the same in quantity. I would not say that phosphorus has no effect on the cold shortness of iron, but I would remark that whatever effect it has is very much modified by the manner of working the iron. And this opinion is sustained by that of others who have had much to do with the working of iron.

SCIENTIFIC INTELLIGENCE.

I. Chemistry and Physics.

1. *Apparatus for illustrating the action of Geysers.*—In J. Müller's Cosmical Physics, 2d ed., p. 386, 1865, Bunsen's theory of the eruption of geysers is illustrated by filling a vertical metallic pipe with water and applying heat to its middle portion and to its lower end. G. Wiedemann believes that this apparatus does not truly represent the actual phenomenon, since it is hard to conceive of the earth's heat being applied in two points; and he has for several years used the following apparatus in his lectures. A Florence flask is provided with a long vertical glass tube drawn out at its upper end into a comparatively small orifice. Another glass tube passes through the cork which closes the mouth of the flask and is led away at an angle to a glass bottle which it enters by a stop cock at the base of this bottle. This bottle is filled with water, the height of the liquid being about at the level of the top of the vertical glass tube which rises from the Florence

flask. The Florence flask filled with water is heated from beneath by a Bunsen burner, and water entering through the stop cock from the glass bottle is raised to the boiling point. The bubbles of steam rise in the vertical tube and the alternations of heating in the flask and changes of level of the water in the vertical tube cause steam and water to issue with violence from the vertical tube. The tube conveying the cold water to the Florence flask should be curved at its lower end to prevent the cold water from striking the hot base of the flask, and the vertical tube should be flush with the bottom of the cork which closes the Florence flask.—*Ann. der Physik und Chemie*, No. 1, 1882, pp. 173–175. J. T.

2. *Electrical resistance of Gases.* — E. Edlund in this paper reviews the results of previous experimenters, especially those of Becquerel and Hittorf. Since it is known that gases differ from solid and fluid bodies in not conducting electricity until the electromotive force acting upon them has reached a certain limit, Edlund shows that the strength of the currents passing through layers of gases cannot be inversely as the resistance of these layers. Becquerel and Hittorf's experiments, however, lead to this erroneous conclusion. Edlund shows that his unitarian theory of electricity is borne out by the behavior of gas in respect to electrical conduction.—*Ann. der Physik und Chemie*, No. 1, 1882, pp. 165–171. J. T.

3. *Dielectric polarization in Electrolytes.* — The phenomenon of electrical conduction through electrolytes has lately been the subject of much investigation, and R. Colley in 1879 stated his conviction that each electrode immersed in the electrolyte acted, with its layers of gases, as a condenser and the intervening liquid as a conductor. Helmholtz in the same year showed that the electrodes placed very near each other acted as a condenser intercalated in a circuit. R. Colley has returned to the question and has investigated the action of a series of substances which pass by degrees from the condition of conductors to indifferent conductors of electricity. Among these substances were benzol, light benzine, heavy benzine, petroleum, olive oil, solution of chloride of iron in benzol, and ether. The results of his investigation confirm the views of Faraday and Maxwell in regard to electrical action in a dielectric. Dielectric polarization exists not only in insulators but also in conductors, and the dielectric constants of electrolytes are of the same order of greatness as those of true dielectrics.—*Ann. der Physik und Chemie*, No. 1, 1882, pp. 94–111. J. T.

4. *Change of temperature due to extension and contraction of metallic wires.*—In works on thermo-dynamics the expression $\vartheta = -\frac{(273+t)\,\alpha P}{A \cdot w \cdot c}$ is given for the change of temperature which results from mechanical strains in wires produced by extension and subsequent contraction. In this expression,

A is the mechanical equivalent of heat,
P the change of tension,
α the coefficient of expansion,
C the specific heat,
W the weight of unit length.

Joule was the first to test the formula and his results did not entirely confirm the theoretical result. Edlund found that the formula led to a value of A=682·7 kilogrammeters, and concludes that the discrepancy must be explained by the presence of internal work in the wire. Rühlman is opposed to the conclusion of Edlund, and does not think it probable that the specific heats of stretched wires can differ essentially from those of unstretched wires. H. Haga has accordingly tested the formula experimentally, measuring carefully the specific heats of the wires. From a steel wire and a German silver wire the values of A were 437·8 and 428·1 respectively, and the author therefore concludes that the formula is correct and the mechanical theory of heat explains the changes of temperature which follow the extension and contraction of wires.—*Ann. der Physik und Chemie*, No. 1, 1882, pp. 1–18. J. T.

5. *Phosphorescence.*—Capt. ABNEY, R.E., at a meeting of the Physical Society, London, Jan. 28, exhibited some experiments upon phosphorescence, and showed that a surface covered with Balmain's luminous paint, which is violet when excited by daylight, becomes more violet when it receives the blue rays of the solar spectrum. The red end of the spectrum, however, extinguished the violet phosphorescence. The light of an electric lamp passed through a sheet of red glass also extinguished the phosphorescence. Capt. Abney believes that there are a series of octaves in the blue end of the spectrum which do not extinguish the violet light. The mean wave length of the rays exciting the phosphorescence was found to be 4300.—*Nature*, Feb. 9, 1882. J. T.

6. *Discussion of the Fourier-Poisson theory of the conduction of heat.*—The principal objections to this theory are the following: 1. It does not take into consideration changes in specific heat. 2. It considers that the conductivity of a metal is independent of its expansion. 3. It considers the exchange of heat between two contiguous bodies proportional to the difference between the temperatures U and U′ of their boundaries. W. Hergesell discusses these objections and thinks that the objections 1 and 3 are not important, and that the points can be embraced by the theory. He shows, in the case of 2, that Fourier's theory demands that if the conductivity is diminished by extension that the flow of heat between like particles is increased (under certain limitations).—*Ann. der Physik und Chemie*, No. 1, 1882, pp. 19–38. J. T.

7. *Spectroscopic observations with monochromatic light.* — M. ZENGER finds that benzine and benzylene in a compound prism together with quartz at an angle of 75° eliminate the extreme red, while pure anethol at the refracting angle of 75° eliminates

the extreme violet. A compound prism or parallelopiped thus formed affords the best means of studying solar protuberances and spots and the reversed lines of the chromosphere. The prism is recommended to the Transit of Venus Commission, since clearness of definition can be obtained under great dispersive and large magnifying power. It also obviates confusion arising from interference bands at the edge of the disk of Venus and the effects of irradiation are obviated. It is also claimed that better solar photographs can be obtained by the use of this prism.—*Comptes Rendus*, Jan. 23, 1882, pp. 155–157. J. T.

8. *On crystals of Potassium Chloride found in the extract of Wormwood or Absinth* (*Artemisia Absinthium* L.; by E. Claassen. (Communicated).—Some time ago I had occasion to make use of the above named extract and was surprised to find in it many perfectly transparent, yellowish, almost colorless crystals of great regularity of form. The largest were about ¼ inch in diameter and weighed 130 milligrams (about two grains). By studying their crystallographic form I found that they belonged to the isometric system; one crystal was a cube with the angles slightly replaced with octahedral planes, the others were combinations of the octahedron and cube, with either a predominant octahedron or with no predominance of either of the forms. Several of the last mentioned crystals show, however, also very nicely and distinctly besides these forms, the icositetrahedron 2-2 (2O2). By chemical analysis it was found, that the crystals were potassium chloride.

This salt is commonly found in cubes, and the uncommon forms described are perhaps due to the presence of organic substances in the wormwood extract.

Cleveland, Jan., 1882.

II. Geology and Mineralogy.

1. *Tides in early Geological time.*—Mr. G. H. Darwin, whose paper on the "Precession of a Viscous Solid" (Phil. Trans., 1879), called out the remarks of Mr. Ball, Astronomer Royal of Ireland, on great tides as a geological agency in early time, states, in Nature for January 5th, his non-concurrence with Mr. Ball in his conclusions. The enormous tides referred to by Mr. Ball, he would locate in pre-geological periods. He had contemplated the possibility of tides two or three times as high as at present in the earliest geological time, and observes that this estimate is probably rather excessive than deficient. As in his former paper, Mr. Darwin states that an increase of rain-fall would be an undoubted result of the conditions, and that this should have had its geological effects. When the tide was 648 feet high the moon was about ten earth's diameters distant, and, Mr. Darwin observes, the earth must have been rotating in about seven hours; the trades would probably have had a velocity of 3¾ their present velocity, and, consequently, vortical storms would have had pro-

digious violence; but these conditions would have favored the making of a large proportion of very coarse-grained sedimentary rocks, contrary to the fact among the earliest of fossiliferous strata.

Tides having twice the height of modern tides (such as existed when the moon was a fifth nearer the earth than now) would have aided much in degradation during Archæan times, by increasing the vertical range of wave and current action and the working force of tidal currents flowing through shallow seas or channels; and this action together with that of erosion resulting from large precipitation, are the chief geological effects over the earth's surface which Mr. Darwin attributes to the cause mentioned. Geologists have here no reason to question Mr. Darwin's judgment. Another effect is also mentioned in the original paper, and referred to here again: that "the ellipticity of figure of the earth must have been continually diminishing, and thus the polar regions must have been ever rising and the equatorial ones falling; but as the ocean always followed these changes, they might quite well have left no geological traces."

2. *Contributions to the History of the Vertebrata for the Lower Eocene of Wyoming and New Mexico, made during* 1881; by E. D. Cope. Read before the Amer. Phil. Soc., Dec. 16, 1881. Cope's Palæontological Bulletin, No. 34.—This paper by Professor Cope treats especially of the Lower Eocene of the basin of the Big Horn River. The upper drainage basin of the Big Horn River, which was examined in 1859 by Dr. Hayden and then pronounced Lower Eocene, was explored in 1880 by a party under J. L. Wortman, sent out by Professor Cope. From the specimens collected, Professor Cope confirms the results of Hayden's survey, concluding that the beds are intermediate between the Bridger and Wasatch beds, they containing genera hitherto regarded as peculiar to each. A lower basin lies on the same river north of the Big Horn Mountains. In 1881, Mr. Wortman, under the same auspices, made further explorations of the region, and obtained numerous extinct vertebrates from the Lower Eocene horizon. These Eocene beds "cannot be less than 4000 feet in vertical depth;" they are now eroded in bad-land style and are under bad-land dryness and other features. They consist of variously colored clays, a large part red, alternating with brown and blue sandstone layers. The species of vertebrate fossils mentioned include fishes, reptiles and mammals. Among the last, twenty-four are described as new species, and some of these are referred to new genera, and they are stated to represent fully the Wasatch fauna with little admixture of earlier or later forms. In place of the characteristic Middle Eocene (Bridger) genera, *Hyrachyus*, *Palæosyops*, *Uintatherium* and the *Tillodonta*, there are *Phenacodus*, *Hyracotherium*, *Coryphodon* and *Tæniodonta*, as in New Mexico; but several genera are, as elsewhere, common to the two horizons, and two species from both, *Hyopsodus paulus* and *H. vicarius*, cannot be distinguished by the parts preserved. The

Big Horn collection is peculiar among those of Wasatch age, in the presence of numerous species of *Phenacodus* and of new and rare species of genera of *Coryphodontidæ*. The new Coryphodonts here described are *Manteodon subquadratus*, *Ectacodon cinctus*, *Coryphodon anax*, *C. repandus*, *C. curvicristis*, *C. marginatus*, *Metalophodon testis*; and besides these, four previously described species occur among the fossils, *C. cuspidatus*, *C. latipes*, *C. simus* and *C. elephantopus*. The collections include the new Artiodactyles, *Mioclænus brachystomus* and *M. etsagicus*, species of the oldest type of Artiodactyle yet described. The first of these species was about the size of the Javan Musk-deer *Tragulus Javanicus*. The characters of the genus are, in general, similar to those of *Dichobune*, even to the presence of the external digits (which are wanting in the Anoplotheriidæ); it differs "in the presence of but one internal tubercle of the superior molars and in the single external tubercle of the superior premolars."

The collections contain two species related to the Lemurs and family *Prosimiæ*, named *Cynodontomys latidens* and *Anaptomorphus homunculus*. The latter is remarkable for the small size of the canine teeth. The genus is nearest to *Tarsius* among the Lemurs. Contrary to the ordinary fact among the early Mammals, the brain and its hemispheres are not at all smaller than those of Tarsius or of the typical modern Lemurs.

3. *Report of the State Geologist of New Jersey*, Professor G. H. Cook, for the year 1881. 108 pp. 8vo, with a colored Geological Map of the State.—This report is devoted chiefly to the subject of New Jersey climate. With regard to the iron-ore districts, largely Archæan, the report gives the welcome information that the survey has had the construction of careful topographical and geological maps of the districts in progress for the two years past, and that the surveys now completed cover nearly three-fifths of the whole. The topography is based on the U. S. Coast Survey triangulation, and elevations will be given by carefully determined contour-lines for every twenty feet. By the aid of the maps, it is expected to make out completely the system in the distribution of the ore-beds so as to be able to point out where they are to be looked for and where not.

4. *Dioptase from Arizona*; by R. C. Hills. (Communicated.)—The rare mineral dioptase has been recently found at the Bon Ton group of mines, near the head of Chase Creek, about nine miles from Clifton, Arizona. It occurs in brilliant crystals lining cavities in what is called locally the "mahogany ore," a dark-brown compact mixture consisting principally of limonite and oxide of copper in varying proportions.

5. *Analysis of a variety of Siderite*; by Edo. Claassen. (Communicated).—This variety of siderite occurs on hematite in the Lake Superior Iron Region, Mich., either in single rhombohedral crystals, or more commonly in crystalline crusts. It is often also found associated with crystals of calcite, which then form on

the hematite a coating covered more or less by the crystals of siderite. The siderite has a light green color, which by exposure of the crystals to air and water turns brown and yellowish-brown. It gives the usual reactions for siderite, and also a slight manganese reaction. It contains

$FeCO_3$	$CaCO_3$	$MgCO_3$	$MnCO_3$	
66·240	28·362	5·391	trace	= 99·993

or

FeO	CaO	MgO	MnO	CO_2	
41·115	15·883	2·567	trace	40·428	= 99·993

According to the above analysis the ratio of $FeCO_3 : CaCO_3 : MgCO_3$ is $= 9 : 4\frac{1}{2} : 1$.

Cleveland, O., Jan., 1882.

III. Botany and Zoology.

1. *The Names of Herbes.* By William Turner, A. D. 1548. *Edited (with an Introduction, an Index of English Names, and an Identification of the Plants enumerated by Turner)*, by James Britten, F.L.S., of the British Museum, London; published for the English Dialect Society, Trübner & Co., 1881. The original full title is "The Names of herbes in Greke, Latin, Duch and Frenche wyth the commune names that Herbaries and Apotecaries vse. Gathered by William Turner." The dedication to the "Duke of Summerset" and preface addressed to him, is "from your graces house at Syon," and from Isleworth, almost under the shadow of Syon House, Mr. Britten issues his editorial preface. Full of instruction and quaint interest is this neat volume, of 134 pages, 8vo. A. G.

2. *A Synopsis of the North American Lichens*, part I, by Edward Tuckerman. Boston: Cassino, 1882. pp. 262, 8vo.— It is a great pleasure to receive this volume, which cryptogamic students have greatly needed and longed for. It is the first part of a work intended to describe all our North American Lichens; and we have here a very important portion of them, namely, the *Parmeliacei*, *Cladoniei* and *Cœnogoniei*, arranged in the manner proposed by Prof. Tuckerman in his *Genera Lichenum*, published in 1872. In an introduction of seventeen pages the author briefly states his views with regard to the nature of Lichens, which he thinks ought, together with Algæ and Fungi, to be still considered as the three classes into which Thallophytes should be divided. With respect to the development of the lichen-thallus he follows in the main the views expressed by Minks in his work on the Microgonidium, and he adopts the account given by Bayerhoffer and Minks of the development of the spermogonium and apothecium. The remainder of the work is a systematic description of the species of the three families named above which occur in Arctic and temperate America. As usual,

Prof. Tuckerman has shown a wise conservatism with regard to the limitation of species, avoiding the excessive multiplications, founded on trivial or accidental distinctions, which characterize the writings of some modern lichenologists. Nor does he to much extent make use of chemical tests for the purpose of distinguishing species, as is done by Nylander and several European botanists, who have pushed this means of diagnosis to an extreme. The descriptions are clear and adequate, without being diffuse, and the botanist and student are now in position to determine North American Lichens with a degree of ease hitherto unknown: for the earlier writings of Prof. Tuckerman, as well as his Lichenes Exsiccati, have long been difficult or impossible to procure, and many of his recent writings are scattered in the proceedings of societies to which the botanical public do not generally have access. It is a rare good fortune for those interested in the study of our Lichens that the pioneer in this department of botany in America should also be the one to publish the first complete account of the species of the United States; for, in addition to his own great experience and protracted researches carried on in this country, Prof. Tuckerman in the very beginning of his botanical career was the personal friend and correspondent of Elias Fries, the most learned and acute lichenologist of his time. He has been able, so to speak, to transmit to us in this country the views of the fathers of the science with regard to North American species and their relations to European forms. In the case of marine algæ, Harvey unfortunately died before any one in this country began really to study that group of plants, and we are left in the dark as to many species about which Harvey alone could have given information. With the death of Curtis, too, students of Fungi lost the only person capable of pointing out with certainty in our fields and woods many of the larger fungi whose determination is now uncertain, but which would have been simple enough to one who had had personal instruction from Curtis.

As the greater part of the more striking Lichens belong to the *Parmeliacei* and the *Cladoniei*, the student will find most of the forms which he is likely to collect described in this first part of the Synopsis, and the determination of them should be by no means difficult, especially as Prof. Tuckerman is not given to excessive splitting up of species. The family of the *Collemiei*, however, must present difficulties even to experts. It seems to us on the whole advisable, in describing the gonidia (*gonimia*) of the genera of this family—entirely regardless of the fact whether one is a believer or disbeliever in the algo-fungal nature of lichens—to retain the names given by algologists to similar forms, as Nostoc-like, Glæocapsa-like, for such names accurately describe the forms present without necessarily committing one as to their nature. It is to be hoped that the learned author will soon give to the public the remainder of the Synopsis, including the very difficult *Lecidea* and *Verrucaria* groups, with which he alone of American botanists is able to cope. With the appearance of

Part II. we shall of course have the index of species and synonyms, of which, in using the present part, we feel the want. As no temporary index is given, we infer that the remainder of the work will soon follow. W. G. F.

3. *Botanische Mikrochemie;* by V. A. POULSEN, translated by CARL MÜLLER.—This little book is to be recommended to all persons in charge of laboratories where botanical work is carried on. It forms a small handy-volume to be kept on one's working table, and gives in a condensed form the reactions used to distinguish vegetable tissues under the microscope, with directions for preparing the various reagents, together with information as to the best way of mounting different botanical objects. The present German translation was made from the original Danish edition, with the sanction of the author, and although condensed in form, it can easily be read by any one having a slight acquaintance with German. We have not yet heard of its appearance in the American book market, but it can be ordered of the publisher, Theodor Fischer of Cassel, for a couple of marks. W. G. F.

4. *Nature and functions of the "Yellow Cells" of Radiolarians and Cœlenterates.*—A paper under this title was read by Mr. Patrick Geddes before the Royal Society of Edinburgh in January last, and an abstract of it is given in Nature of January 26, from which we take the following notes. The "Yellow Cells" referred to, first so named by Huxley, occur abundantly in most Radiolarians. They have a well-defined nucleus and multiply rapidly by division. Hæckel made them probably secreting cells, or digestive glands in the simplest form. Cienkowski, in 1871, pronounced them parasitic algæ, and showed that they survived and multiplied after the death of the Radiolarians. Similar cells were later found in Velella, the oral region of some jelly-fishes, and in Anemones. The author in 1878 collected and analyzed the gas given out in direct sunlight by a Planarian, *Convoluta Schultzii*, and found it to be one-half oxygen, and further detected starch in the green cells. At Naples, in October, 1881, he confirmed the conclusions of Cienkowski as to the yellow cells of Radiolarians, and found starch invariably present. The same results were obtained with Velella, sea-anemones and jelly-fishes. On exposure to sunshine, a Velella gave out bubbles of gas which proved to be about one quarter oxygen, and similarly *Anthea cereus* gave gas which was 32 to 38 per cent oxygen; and among other sea-anemones those specimens in which the yellow cells existed gave a like result. A white Gorgonia contained the yellow cells and afforded the usual results, while a red variety of the same species failed of the cells and of the consequences. From the observations, Mr. Geddes concluded that the coloring matter, described as chlorophyl by Lankester, was derived from these endodermal algæ; also that the algæ aided the animal in its functions by supplying material for digestion when the alga dies; by removing carbonic acid and nitrogenous waste produced by the animal, thus performing an "intracellular renal function;" and by

supplying oxygen directly to the "surrounding animal protoplasm," "foreign chlorophyl thus performing the respiratory function of native hæmoglobin." Specimens of Anthea placed in the sunlight began at once to wave their arms as if pleasantly stimulated by the oxygen developed in the tissues. The fact of the absorption of the oxygen was apparently proved by experiment. A green alga (*Ulva*) gave, in the sunlight, gas containing 70 per cent of oxygen; a brown alga (*Haliseris*) 45 per cent; Diatoms, 42 per cent; while with animals containing the alga, as above shown, the amount of oxygen in the gas evolved is less than this and in all the trials except those with species of Ceriactis, much less. Mr. Geddes named the alga *Philozoon*, and distinguished four species, *P. radiolarum*, *P. siphonophorum*, *P. actinarum*, *P. medusarum*.

Mr. Geddes closes with the observation that the nearest analogue to the above described "partnership is to be found in the vegetable kingdom, where, as the researches of Schwendener, Bornet and Stahl have shown, we have certain algæ and fungi associating themselves into the colonies we are accustomed to call lichens, so that we may not unfairly call our agricultural Radiolarians and Anemones animal lichens."

5. *Ueber das Zusammenleben von Thieren und Algen.*—In connection with the preceding account of the paper of Mr. Geddes, attention should be called to a paper by K. Brandt, read before the "Gesellschaft Naturforschender Freunde" of Berlin, Nov. 15th, 1881. In the paper which bears the title above mentioned, Brandt gives the results of his investigations on the colored bodies in *Hydra*, *Spongilla*, a fresh-water *Planaria* and several Infusoria. The green bodies were obtained free by crushing the animals mentioned and were examined under a high power with the following result. Each body consisted of a colorless protoplasmic mass in which was imbedded a green bowl-shaped substance which proved on spectroscopic examination to be true chlorophyl. The protoplasm also showed a distinct nucleus. Brandt concludes that the green bodies are independent unicellular organisms to be classed with algæ and he describes the new genus *Zoochlorella;* one species, *Z. conductrix*, occurring in *Hydra* and *Z. parasitica* in *Spongilla*. He includes the yellow cells of the *Radiolariæ*, *Hydrozoa*, and *Actinæ* in a new genus *Zooxanthella*. The green bodies obtained free by crushing *Hydra* and *Spongilla* did not die but continued to live for weeks on slides, and when exposed to the light starch was formed in them. That the species of *Zoochlorella* were not only morphologically but physiologically distinct was shown by this fact. The green bodies of *Spongilla* when absorbed by chlorophyl-free Infusoria were either digested or discharged unchanged, while the green bodies obtained from *Hydra* were absorbed and remained unchanged. Brandt concludes as follows: "So long as they are alive the *Zooxanthellæ* and *Zoochlorellæ* wholly support the animals in which they live. So long as the animals contain few or no green or yellow cells

they nourish themselves like true animals by the absorption of fixed organic material; as soon, however, as they contain a sufficient amount of Algæ they are nourished like true plants by the assimilation of inorganic material. Whenever in consequence of a lack of light the Algæ cannot perform their proper function they (the host-animals) must again nourish themselves like animals." W. G. F.

Botanical Necrology.

Thomas Potts James.—With sorrow we have to announce the decease of one of our oldest botanists, which has taken place just when the fruits of long and laborious studies were nearly ready to be gathered. To Bryologists Mr. James' intimate acquaintance with mosses and his devotion to their investigation were not unknown. But the publication of the Manual of North American Mosses, by the two veterans, Lesquereux and James, for which we were waiting, would at once have resulted in wide appreciation of devoted services. Let us hope that the preparation of this long-desired volume is so far advanced that his equally venerable associate may soon complete and give it to the world. Nine years ago we rejoiced in having four devotees to Bryology, giving to this department unusual strength. In the spring of 1873, the unequalled leader and founder of the critical study of mosses in this country, Sullivant, was taken away, the keen-sighted Austin followed in the spring of 1880; and now, the present bereavement leaves Leo Lesquereux alone.

Mr. James died, at his residence in Cambridge, on the 22d of February, 1882, in the seventy-ninth year of his age. In his usual health he had attended the Ash Wednesday morning service and had resumed his daily task with the microscope, on rising from which in the afternoon, the rupture of a blood vessel of the brain brought on a paralysis of one side and a coma, to which he succumbed in a few hours.

Mr. James came of a notable stock of Penn colonists. His maternal ancestor, from whom his baptismal name was derived, was Thomas Potts, of Colebrook Dale, and his great great-grandfather, David James, immigrated from Radnorshire, in 1682, and settled at Radnor, Pennsylvania, near Philadelphia, where the subject of this notice was born, September 1, 1803. Circumstances prevented him obtaining a collegiate education at Princeton, for which he was preparing, and directed him to business affairs. He became a wholesale druggist, and carried on the business with success in Philadelphia for forty years, devoting the while his leisure to botany, for which he had a fondness from early youth. In December, 1851, he married Isabella Batchelder, of Cambridge, Mass., a lady of similar scientific tastes and of varied accomplishments, who with their four children survives; and in 1869 the family removed to Cambridge, where the remainder of his life was passed. In Philadelphia he was an active member of the leading scientific societies, especially of the old-

est one in the country, viz: the American Philosophical Society; of the Horticultural Society he was for twenty-five years the efficient Secretary; of the American Pomological Society he was one of the founders and for twenty-seven years the treasurer; of the American Pharmaceutical Society he was for many years an active associate; and after removing to Cambridge he was chosen a Fellow of the American Academy of Arts and Sciences, and a member of the Boston Society of Natural History. In Botany, after having become familiar with the phænogamous vegetation of the neighborhood of Philadelphia, he had the good sense to take up a specialty, and the perseverance to follow it up; and so he became a proficient and an authority in Bryology. His first published paper upon Mosses was contributed to the Proceedings of the Philadelphia Academy of Natural Sciences in the year 1854. Others appeared in the Transactions of the American Philosophical Society, and in the Proceedings of the American Academy of Arts and Sciences (the latter in connexion with Mr. Lesquereux); and to the Botany of Clarence King's Exploration on the Fortieth Parallel, he contributed the elaborate article on the Musci, in which several new species were characterized. After these successful essays, Mr. James was ready for more systematic work in his chosen field. The great desideratum was, and is, a Moss-flora of the United States. Mr. Sullivant, having completed his *Icones Muscorum*, was about to turn his attention to this greatly needed manual, in conjunction with his associate, Lesquereux, when he was taken from us. His surviving associate, deeply engaged in fossil botany, turned to Mr. James, who was able to give more time to this work; and our lamented friend, at a time of life when most men court repose, consented to joint authorship, and its onerous tasks, from a simple regard to the interests of his favorite science, and in view of the urgent needs of a rising school of botanists and amateurs, to whom mosses were becoming attractive. To this task Mr. James gave himself with single and untiring devotion. Owing to the state of his associate's eye-sight, the whole work of microscopical analysis has fallen upon Mr. James, has been most laboriously and conscientiously performed, and had been brought near to a conclusion when the pen and pencil dropped from his hand. Those who in future use the Manual of North American Mosses,—which we trust Mr. Lesquereux may finish and bring out at an early day,—and also the American Bryologists of the coming generation, building upon the foundations thus laid, will have good reason to honor the memory of these faithful pioneers who have opened and smoothed the way to a fascinating study. And the memory of Mr. James, the kindly, simple-hearted, devout, gentle man—admirable in every relation of life—is a prized possession of a large circle of acquaintances and friends.

Joseph Decaisne, the most eminent botanist in France, the director of the *Jardin des Plantes* ever since the death of Mirbel, the Professor of Culture in the Museum of Natural History at

Paris for more than forty years, died on the 8th of February, 1882, in the 75th year of his age. He was born at Brussels, March 11, "1809," according to Pritzel's Thesaurus; but this must be a misprint for 1807. He must have been a very young man when first attached to the Garden of Plants, it is said as a gardener. For it was there that he began the series of his published contributions to botanical science, in the year 1831; and he soon became *aide naturaliste* in that department of the Museum. He was elected into the Institute in 1847, taking the place in the Academy of Sciences vacated by the death of Dutrochet. In 1842, after the death of Guillemin, he was associated with Adolphe Brongniart in the editorship of the botanical part of the *Annales des Sciences Naturelles*, and has been sole editor since the death of Brongniart. In 1858 he began the publication of that great work, *Le Jardin Fruitier du Museum*, which was only a few years ago brought to a close. He had already produced (in 1837) his capital memoir on the anatomy and physiology of the Madder plant and the development of its coloring matter; also (in 1839), his investigation of the organization of the sugar-beet; his well-known paper upon the Mistletoe; his classical memoir on the *Lardizabaleæ;* his essay on the classification of *Algæ* and his restoration of the Corallines to the vegetable kingdom, not to speak of other papers and separate publications, all bearing the impress of his sincere and faithful workmanship. The same may be said, somewhat qualifiedly, of his elaboration of two orders for DeCandolle's Prodromus, the *Asclepiadeæ* and the *Plantaginaceæ;* but Decaisne worked slowly and with great painstaking, and was probably unable to give the requisite time for such monographs. He appears to full advantage only in separate and leisurely prosecuted investigations. Yet his work on the *Pomaceæ* might have been in truer perspective—according to the general judgment of botanists—if he had not so concentrated his attention upon this group as to view comparatively small differences under exaggerated proportions. But, withal there is much to be said for his decided opinion that the Apple, the Pear, and the Mountain Ash are of distinct genera. Decaisne was a capital draughtsman, and to his gifts in this regard we are much indebted for the *Traité Général de Botanique, descriptive et analytique* which he published in conjunction with LeMaout, in 1868, a veritable treasure to teacher and pupil. The older botanists will sadly feel the loss of Decaisne, and the *Jardin des Plantes* will be much changed to them, now that the last representative of the *régime* of Adrien de Jussieu and Brongniart disappears in the decease of this accomplished botanist and most amiable and excellent man.

To supply the omitted necrology of 1880 and 1881, the following memoranda are inserted:

Coe F. Austin, of Closter, New Jersey, died March 18, 1880, at the age of 49. He was a keen bryologist, and was preparing a manual of North American *Hepaticæ*.

Wm. Philip Schimper, leading bryologist as well as a foremost vegetable paleontologist, died March 20, 1880, at the age of 72. His rich and important herbarium of Mosses has been acquired by the Herbarium of the Royal Gardens at Kew.

Nils J. Andersson, the monographer of *Salix*, died at Stockholm, March 27, 1880, in the 60th year of his age. He had been in retirement for several years on account of infirm health.

General Wm. Munro, the learned agrostologist, to whom we looked for a *Species Graminum* (which was to be published in DeCandolle's Monographiæ), died at his residence near Taunton, England, January 29, 1880, at the age of 64; a sad loss.

Dominique Alexandre Godron, one of the authors of the *Flore de France*, died at Nancy, August 16, 1880, in the 74th year of his age.

S. B. Mead, one of our older local botanists, of much activity in former years, died at his home in Augusta, Illinois, November 11, 1880, at a good old age.

W. Lauder Lindsay, a learned lichenologist, and a writer upon the botany of New Zealand, died in November, 1880, at the age of 52.

Ernst Hampe, a veteran bryologist, died at Helmstedt, near Blankenburg in Hannover, where he had so long resided, November 23, 1880, at the age of 85. It has been said of him that when he began the study of Mosses, the 931 species enumerated by Bridel were all that were then known; in 1851, the number had risen to 2303, and they were estimated at 6000 at the time of his death. His berbarium has been acquired by the British Museum.

Alphonso Wood, the well-known author of some very popular botanical text-books and of one or two botanical papers, an enthusiastic devotee in botany, died at his residence in West Farms, New York, January 4, 1881, in the 71st year of his age. The Bulletin of the Torrey Club, of which Professor Wood was for many years a member, has a good biographical notice of him, vol. viii, p. 53–56.

Gottlieb Ludwig Rabenhorst, a mycologist of note, who has published very extensive and valuable *exsiccatæ* of cryptogamous plants, died near Meissen in Saxony, April 24, 1881, at the age of 74.

Matthias Jacob Schleiden, whose name and position in botany were very prominent forty years ago, when his text-book on scientific botany appeared, but who had so completely subsided from scientific pursuits that few were aware of his actual existence, died at Frankfort on the Maine, June 23, 1881, at the age of 77. The proposition that all vegetable tissues are formed of cells, which was fully announced by Mirbel, was made familiar by Schleiden. His countryman, early transferred to Belgium,

Theodore Schwann extended the proposition to animal tissues, and appears to have originated the idea that animal and vegetable cells, as the structural and physiological units of organic nature, were identical. Although like Schleiden, Schwann's name had

become one of the past, it appears that he actively fulfilled the functions of his professorship, first at Louvain (to which he was called in 1839, in his twenty-eighth year), and afterwards at Liége, quite down to the close of his life, early in the present year. His researches, published in 1839, upon which his scientific fame rests, appear to have been essentially his last, as well as his first contribution to science. It was a most noteworthy contribution, however, if it contained, as is said, the announcement that animal tissues originate in cells, that Bacteria are the cause of putrefaction, and that alcoholic and analogous fermentations are produced by plants. A. G.

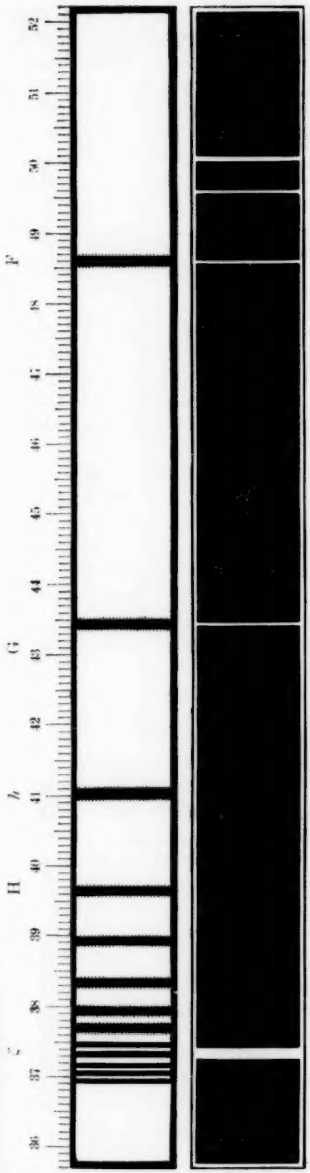
6. *The Census Report on the History and Present Condition of the Fishing Industries;* prepared under the direction of Professor S. F. Baird, U. S. Commissioner of Fish and Fisheries, by C. Brown Goode, Assist. Director U. S. Nat. Mus. *The Seal-Islands of Alaska;* by Henry W. Elliott. 176 pp. 4to, with 29 plates and other illustrations.—This volume treats of the Fur Seal Islands (the Pribylov), and the seals, with great fulness, from all practical and scientific points of view. Its plates illustrate the seal in various attitudes and conditions, singly, and in indefinite multitudes along the shores, and also the walrus, sea-lion, and birds of the region; and contains views of the people, and their houses, the coast scenery, and maps of the sealing region. The number of seal skins taken to market from the Pribylov Islands (St. George and St. Paul), in 1880, was 100,000. These islands lie in the Bering Sea (not Behring, remarks the author), between latitudes 56° and 57° and the meridians 168° and 170°, about 200 miles west of Cape Newenham. They are volcanic in origin, but have no active fires.

IV. Astronomy.

1. *Micrometrical Measurements of 455 Double Stars*, 1879–80; Publications of the Cincinnati Observatory, No. 6, University of Cincinnati. Cincinnati, 1882. Large 8vo, pp. 69.—The sixth contribution to the Astronomical literature of double stars is alike creditable to the University of Cincinnati, Professor Stone personally, and the printer. Perhaps it would be not an easy matter to find a better illustration of the wisdom in confining the staff of a moderately equipped observatory to such observations as do not imply expensive or tedious reduction, than is given in the consecutive publications of the observatory since it resumed its activity under Professor Stone. The observations seem to be made with continuous care, according to a fixed system, and with due regard to personal and systematic errors. The results are given in excellent shape for present reference or for the examination of them at a future time for errors too subtle to be now suspected. L. W.

2. *Names of small Planets.*—The following names have been given to recently discovered planets:

212 Medea,	218 Bianca,
216 Cleopatra,	219 Thusnelda.



3. *Photograph of the Spectrum of the Great Nebula in Orion;* by WILLIAM HUGGINS. (Communicated by the Author.)—On Tuesday night, 7th March, I obtained a photograph of the spectrum of the nebula in Orion, with an exposure of 45 minutes. The spectroscope and special arrangements attached to the Cassegrain telescope of 18 inches aperture were the same as were employed in obtaining the photographic spectra of the stars. My former researches showed that in the *visible* spectrum of the gaseous nebulæ four bright lines can be seen. The strongest, coincident with the less refrangible component of the brightest double line of the nitrogen spectrum, has a wave-length 5005. The next line is at λ 4957. The other two lines coincide with Hβ (F) and Hγ of the hydrogen spectrum.

On the photographic plate these four lines can be seen, but in addition there is a strong line in the ultra-violet at the position of λ 3730, or very nearly so, as the wide slit does not permit of quite the same high degree of accuracy of measurement as was possible in the case of the spectra of the stars.

It is very probable that this new line coincides with the line ζ in the typical spectrum, as shown by my photographs, of the brightest white stars. For the convenience of comparison I have placed this typical spectrum by the side of the spectrum of the nebula. I could not be certain if any *faint* lines are present between Hγ and the new line at λ 3730, and also beyond this line. I hope by longer exposures and more sensitive plates to obtain information on these points.

There was a faint continuous spectrum on the plate, which was probably due to stellar light. The bright stars of the Tra-

pezium were placed close outside the slit, so that the light of the brightest part of the spectrum might pass in to form the spectrum.

4. *Linear Associative Algebra;* by BENJAMIN PEIRCE, with notes and addenda by C. S. PEIRCE, son of the author.—This work, which the distinguished author always regarded as his best contribution to science, was published in his lifetime only by a limited edition in lithograph for personal distribution. Notwithstanding its value it has been therefore difficult for mathematical students to have access to it. Its reproduction in the American Journal of Mathematics is a real boon to science. H. A. N.

V. MISCELLANEOUS SCIENTIFIC INTELLIGENCE.

1. *The Voyage of the Vega round Asia and Europe, with a Historical review of previous journeys along the North Coast of the Old World;* by A. E. NORDENSKIÖLD. Translated by ALEXANDER LESLIE. 756 pp. 8vo, with numerous maps and illustrations and fine steel portraits. London and New York, 1882. (Macmillan & Co., N. Y.)—This large volume is one of great interest to all readers. It is a contribution to the subjects of ancient as well as modern geography, ancient explorations of the northern regions as well as the most recent, old maps and new, old style of ship building in the north and the construction of the Vega, the deep-sea life to some extent and that of the shores and the land, with reproductions of quaint old pictures of polar bears, the walrus and other Arctic species and finished sketches illustrating these species, and also to ethnological, linguistic and other departments, besides giving an account of the incidents, sights and discoveries of the voyage. The portraits include an excellent one of Nordenskiöld.

2. *Guides for Science Teaching*, issued under the auspices of the Boston Society of Natural History. Ginn, Heath & Co., Boston, Publishers. 1879 to 1881.—These small primers, of 24 to 64 pages each, are by authors well versed in the several subjects treated, and will be found of much value to teachers and learners. The titles of the seven before us are: About Pebbles, by ALPHEUS HYATT; Concerning a few Common Plants, by G. L. GOODALE; Commercial and other Sponges, illustrated by several plates, by A. HYATT; A First Lesson in Natural History, by Mrs. AGASSIZ; Common Hydroids, Corals and Echinoderms, by A. HYATT, illustrated by several plates; The Oyster, Clam and other Common Mollusks, by A. HYATT, well illustrated by plates; Common Minerals and Rocks, by WM. O. CROSBY.

3. *Potassium permanganate.*—Dr. J. B. DELACERDA, after careful experimenting, has reached the conclusion that this salt is an antidote to the poison of the Cobra; and that it acts through the disengagement of oxygen as a consequence of its decomposition in the system.

4. *Index to the Reports of the Chief of Engineers and the officers of the Corps of Engineers, United States Army, upon*

Works and Surveys for River and Harbor Improvement, 1866–1879.—Compiled under the direction of Maj. HENRY M. ROBERT, Corps of Engineers, by L. Y. SCHERMERHORN, C.E., S. O. L. POTTER, A.M., M.D., and others. 624 pp. 8vo. Washington, 1881.—The reports of the Engineer Department embrace so wide a range of facts and of careful investigations as regards all scientific and practical questions connected with rivers, lakes, harbors, canals, bridges, improvements by levees, dikes and other means, and the materials and methods of construction, and contain so important contributions to the sciences of hydraulics, dynamical geology and geography, made over the breadth of the American Continent, that this Index volume is one of great value and very general interest. Besides this, they are full of local details as to surveys and improvements in all parts of the country.

5. *Earthquake of March* 4, 1881, *in Ischia.*—The first number of "Humboldt," the new German scientific monthly published by Enke at Stuttgart, opens with a paper by Dr. A. v. Lasaulx, "Das Erdbeben von Casamicciola auf Ischia." The author argues that as the shock was only slightly felt on the island of Procida and not at all on the opposite mainland, the origin must have been at a less depth than the sea-bottom of the intervening strait. And since the district most violently affected is bounded by an oval whose long diameter is east-west and therefore not radial to the volcanic center of the island, which lies to the south, the cause is not to be sought in the volcanic activity of this Mount Epomeo. He attributes the earthquake to subsidence of deep-lying strata, due to hollows caused by removal of material in solution by the numerous hot springs. C. G. R.

6. *The Swiss Seismological Commission.*—We gather from Comptes Rendus (Dec. 26, 1881) and from Nature (Jan. 12, 1882) some particulars in regard to the work of this Commission since it began operations in the fall of 1879. Its members have published a number of papers, which have been recently presented to the French Academy by M. J. L. Soret. Among these are: 1. Note on Earthquakes and their scientific study, by M. Heim; 2. Note on the Seismometric Observations in Switzerland, by M. Forster; 3. Report on the Earthquake of Dec. 30, 1879, by M. Forel; 4. Report on the Earthquakes from November, 1879, to December, 1880, by M. Heim; 5. Note on the same period, by M. Forel; 6. Note on the Earthquakes of November, 1881, by M. Forster. M. Heim writes in German, the others in French, and the first paper is translated into French by M. Forel. In presenting these papers to the Academy, M. Soret cited a number of interesting facts from which we take only two. Some shocks, as those of July 4, 1880, and March 3, 1881, occurred almost simultaneously throughout Switzerland and Northern Italy. These and others appeared also to have spared a considerable part of upper Savoy; being felt, for example, with considerable intensity on the north bank of Lake Geneva, and but slightly on the southern shore, although this was nearer the probable source

of the disturbance; as if this district were, from the seismic point of view, under conditions different from those which affect adjacent cantons. The month of November, 1881, was also marked by special seismic phenomena. According to a note by M. Forel in "La Suisse Liberale" of Dec. 14, 1881, twenty-nine distinct shocks of earthquake had occurred within the month, and of its thirty days there were only thirteen on which an earthquake did not occur in some part of Switzerland. They were traceable to three principal centers; the first, at the commencement of the month, in the Pays d'Enhaut and the Vaudoise Alps; the second in Eastern Switzerland; the third, at the end of the month, in the lower Valais, from Martigny to Lake Geneva.

C. G. R.

7. *Glacier scratches in the Catskills.*—Dr. Julien, in the Transactions of the New York Academy of Sciences, vol. i, no. 2, states that he has found no *glacial scratches* near the Clove above 2,900 feet, the highest observed occurring on the "High Ledge," Parker Mountain, at 2,874 feet, and on the southeast slope of Round Top, at 2,871 feet; the direction of the former S. 18° W., magnetic; of the latter S. 35° E. He remarks that the highest scratches observed in the Catskills occur on Overlook Mountain, at an elevation of about 3,100 feet, showing that the ice surface was at least 3,200 over this part of the Catskill region. He concludes that there were two movements over the region—the movement of the Continental glacier southeastward, and that of the Hudson River valley, southward.

8. *Geological Survey of Great Britain.*—Professor ARCHIBALD GEIKIE has been appoined Director-General of the Geological Surveys of Great Britain and Ireland and Director of the Museum of Practical Geology in London.

The Constants of Nature, Part V: A recapitulation of the Atomic Weights, by F. W. CLARKE, Prof. Chem. and Phys. Univ. Cincinnati. Smithsonian Miscellaneous Contributions, No. 441. 280 pp. 8vo. Washington, 1882.

Fifty Years of Science, being the Address delivered at York to the British Association, August, 1831, by Sir JOHN LUBBOCK, President of the Association. 90 pp. 8vo. London, 1882. (MacMillan & Co.)—The extracts from this address in a former volume of this Journal are enough to create a desire for this volume which gives it entire.

OBITUARY.

Sir CHARLES WYVILLE THOMSON died on the 12th of March, at the age of fifty-two. He was born at Bonsyde, Linlithgowshire, on the 5th of March, 1830. His exploring expeditions in the Lightning, Porcupine and Challenger, in which the "depths of the sea" in the Atlantic and around the world were investigated with remarkable success and multitudes of new discoveries, have made his name familiar to the people of all civilized lands. The publications of his last expedition are still in progress. After graduating at the University of Edinburgh, he was appointed, in 1850, Lecturer on Botany in King's College, Aberdeen, and, in 1870, Regius Professor of Natural History in the University of Edinburgh. His so early departure is greatly to be deplored.